A First Course in Digital Communications

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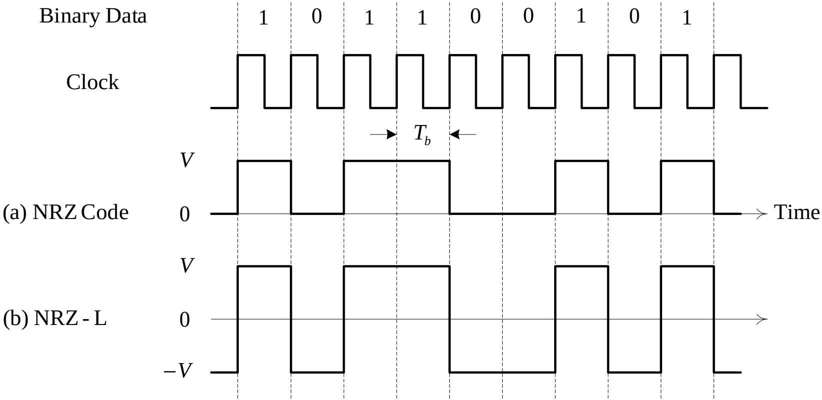
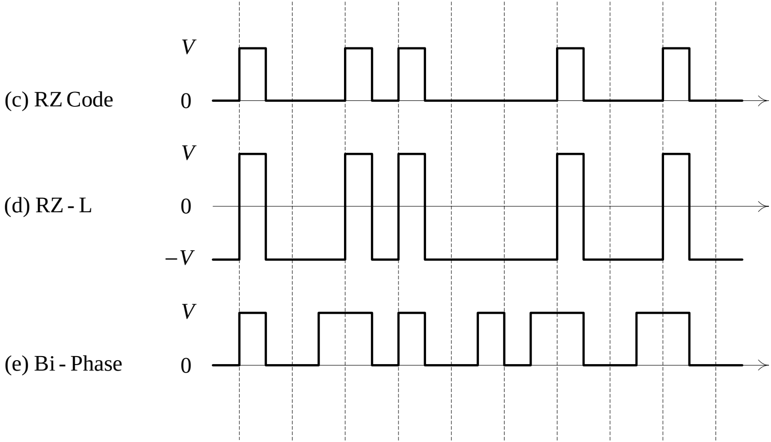
Introduction

- Bits are mapped into two voltage levels for direct transmission without any frequency translation.
- Various baseband signaling techniques (line codes) were developed to satisfy typical criteria:
  1. *Signal interference and noise immunity*
  2. *Signal spectrum*
  3. *Signal synchronization capability*
  4. *Error detection capability*
  5. *Cost and complexity of transmitter and receiver implementations*
- Four baseband signaling schemes to be considered: nonreturn-to-zero-level (NRZ-L), return-to-zero (RZ), bi-phase-level or Manchester, and delay modulation or Miller.

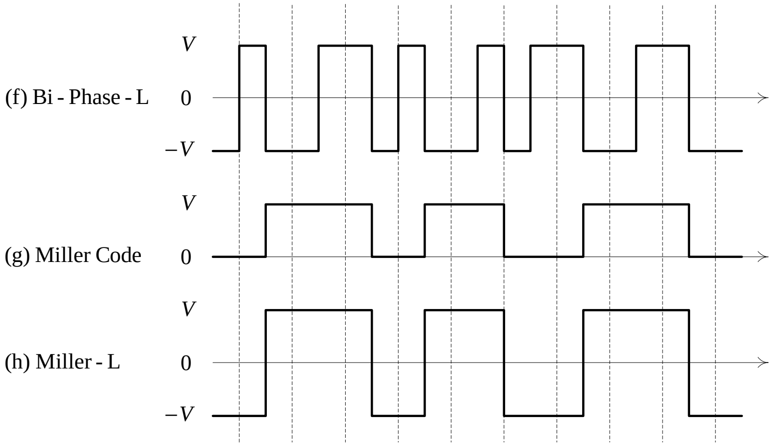
Baseband Signaling Schemes I



Baseband Signaling Schemes II



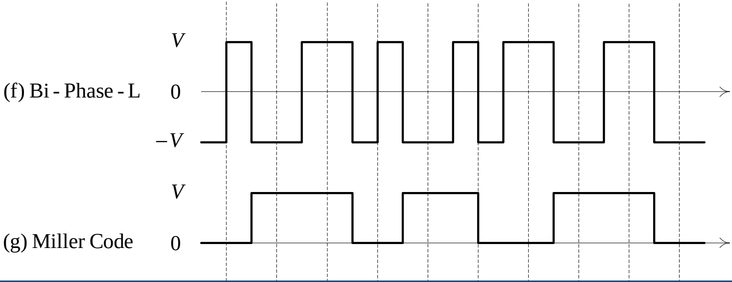
Baseband Signaling Schemes III



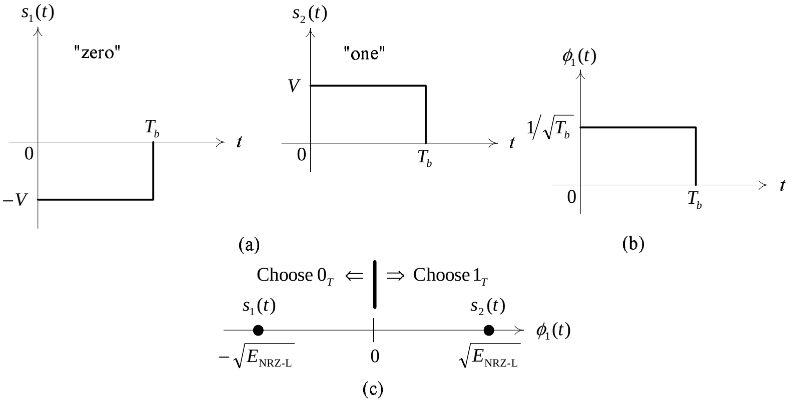
Miller Code

Has at least one transition every two bit interval and there is never more than two transitions every two bit interval.

- Bit “1” is encoded by a transition in the middle of the bit interval. Depending on the previous bit this transition may be either upward or downward.
- Bit “0” is encoded by a transition at the beginning of the bit interval if the previous bit is “0”. If the previous bit is “1”, then there is no transition.

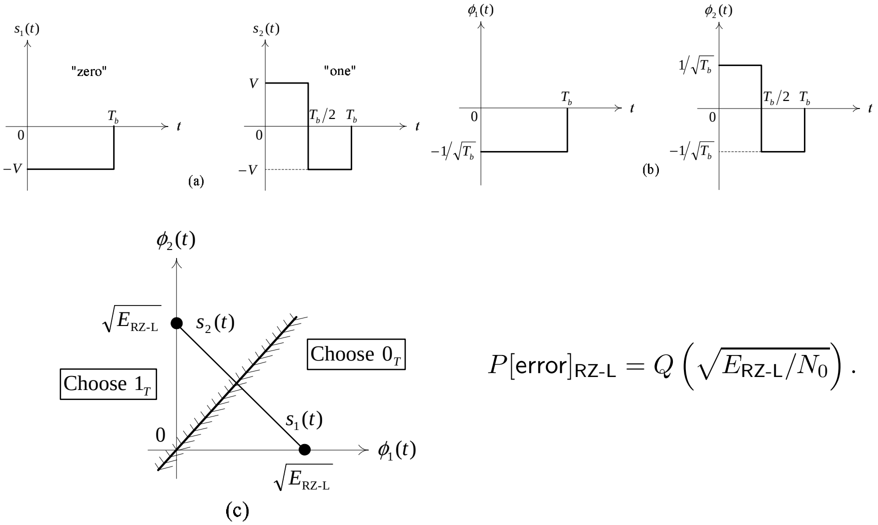


NRZ-L Code



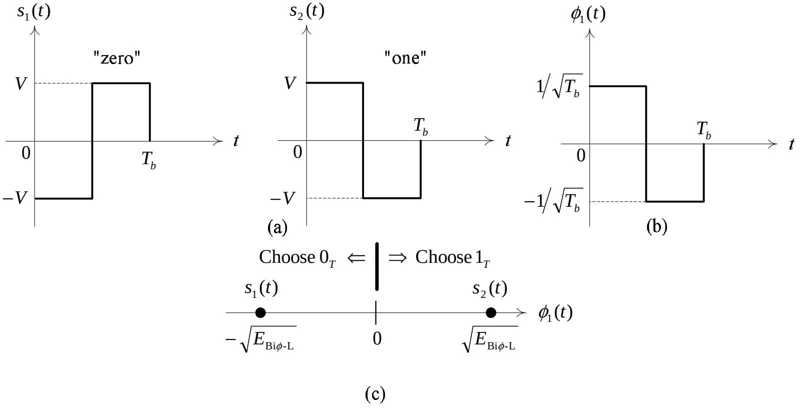
$$P[\text{error}]_{\text{NRZ-L}} = Q\left(\sqrt{2E_{\text{NRZ-L}}/N_0}\right).$$

RZ-L Code



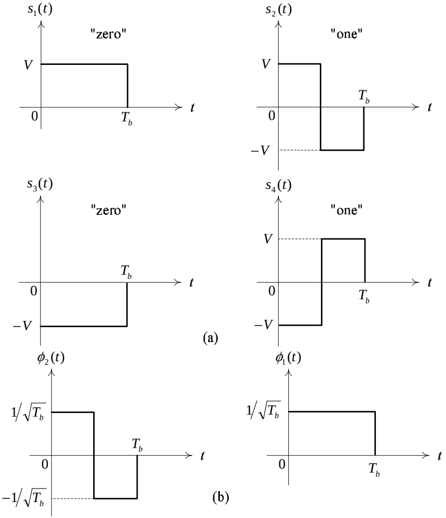
$$P[\text{error}]_{\text{RZ-L}} = Q\left(\sqrt{E_{\text{RZ-L}}/N_0}\right).$$

Bi-Phase-Level (Biϕ-L) Code

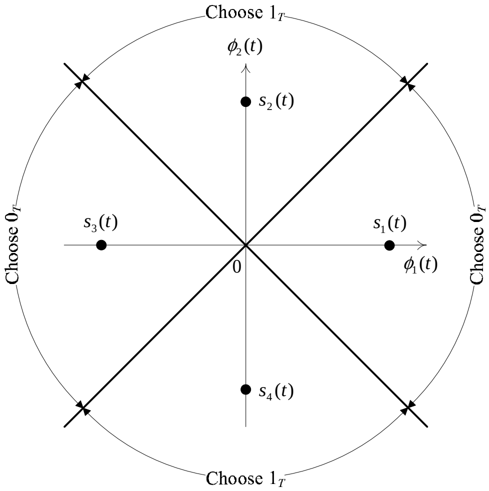


$$P[\text{error}]_{\text{Bi}\phi\text{-L}} = Q\left(\sqrt{2E_{\text{Bi}\phi\text{-L}}/N_0}\right).$$

Miller-Level (M-L) I

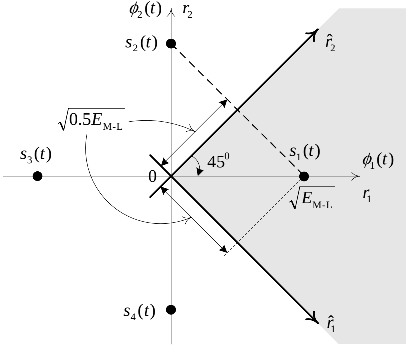


Miller-Level (M-L) II



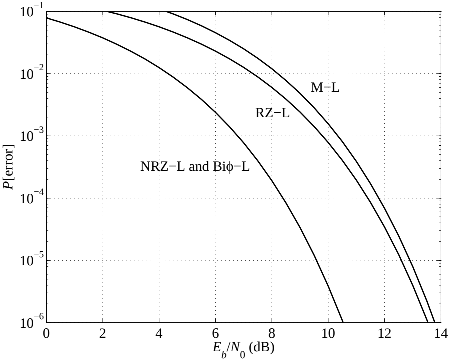
(b)

Miller-Level (M-L) III



$$P[\text{error}]_{\text{M-L}} = 1 - \left[1 - Q\left(\sqrt{E_{\text{M-L}}/N_0}\right)\right]^2$$

Performance Comparison



$$E_{\text{NRZ-L}} = E_{\text{RZ-L}} = E_{\text{Bi}\phi\text{-L}} = E_{\text{M-L}} = V^2 T_b \equiv E_b \text{ (joules/bit)}.$$

$$P[\text{error}]_{\text{NRZ-L}} = P[\text{error}]_{\text{Bi}\phi\text{-L}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right),$$

$$P[\text{error}]_{\text{RZ-L}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right), \quad P[\text{error}]_{\text{M-L}} \approx 2Q\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Optimum Sequence Demodulation for Miller Signaling

- Sequence demodulation exploits memory in Miller modulation.
- Example: The four Miller signals have unit energy and the projections of the received signals on to $\phi_1(t)$ and $\phi_2(t)$ are

$$\left\{r_1^{(1)} = -0.2, r_2^{(1)} = -0.4\right\}, \left\{r_1^{(2)} = +0.2, r_2^{(2)} = -0.8\right\},$$
$$\left\{r_1^{(3)} = -0.61, r_2^{(3)} = +0.5\right\}, \left\{r_1^{(4)} = -1.1, r_2^{(4)} = +0.1\right\}.$$

Transmitted signal	Distance squared			
	$0 \rightarrow T_b$	$T_b \rightarrow 2T_b$	$2T_b \rightarrow 3T_b$	$3T_b \rightarrow 4T_b$
$s_1(t)$	1.6	1.28	2.8421	4.42
$s_2(t)$	2.0	3.28	0.6221	2.02
$s_3(t)$	0.8	2.08	**0.4021**	**0.02**
$s_4(t)$	**0.4**	**0.08**	2.6221	2.42

$\{s_4(t), s_4(t), s_3(t), s_3(t)\}$ is not a *valid* transmitted sequence!

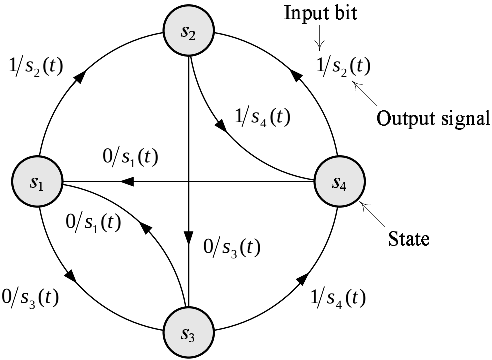
- Assume that a total of n bits are transmitted. Each n-bit pattern results in a transmitted signal over $0 \le t \le nT_b$.
- Denote the entire transmitted signal over the time interval $[0, nT_b]$ as $S_i(t)$, $i = 1, 2, \ldots, M = 2^n$.
- Write $S_i(t) = \sum_{j=1}^{n} S_{ij}(t)$ where $S_{ij}(t)$ is one of the four possible signals used in Miller code in the bit interval $[(j-1)T_b, jT_b]$ and zero elsewhere.
- Received signal over $[0, nT_b]$ is $r(t) = \sum_{j=1}^{n} r_j(t)$ where $r_j(t) = r(t)$ in the interval $[(j-1)T_b, jT_b]$ and zero elsewhere.
- Distance (squared) from $S_i(t)$ to $r(t)$:

$$
\begin{aligned}
d_i^2 &= \int_0^{nT_b} [r(t) - S_i(t)]^2 \mathrm{d}t = \sum_{j=1}^{n} \int_{(j-1)T_b}^{jT_b} [r_j(t) - S_{ij}(t)]^2 \mathrm{d}t \\
&= \sum_{j=1}^{n} \left[\left(r_1^{(j)} - S_{i1}^{(j)} \right)^2 + \left(r_2^{(j)} - S_{i2}^{(j)} \right)^2 \right].
\end{aligned}
$$

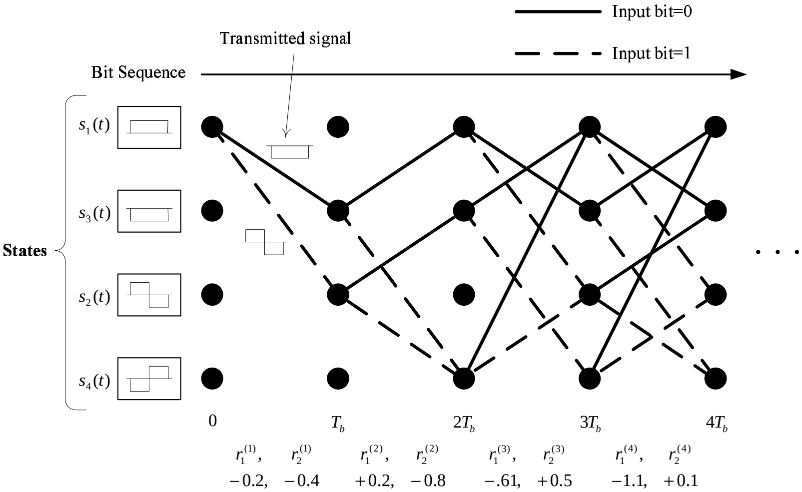
- Direct evaluation of the $M = 2^n$ distances is impossible!

State Diagram

State of a system: Information from the past we need at the present time, which together with the present input allows us to determine the system's output for any future input.



Trellis Diagram



Viterbi Algorithm

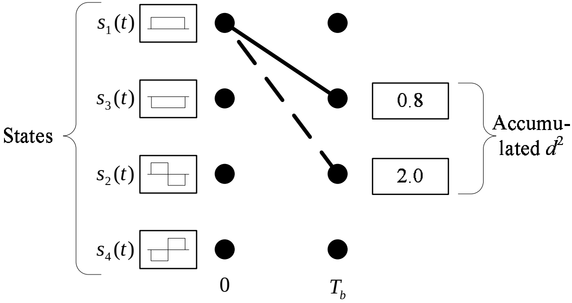
Step 1: Start from the initial state ($s_1(t)$ in our case).

Step 2: In each bit interval, calculate the *branch metric*, which is the distance squared between the received signal in that interval with the signal corresponding to each possible branch. Add this branch metric to the previous metrics to get the *partial path metric* for each partial path up to that bit interval.

Step 3: If there are two partial paths entering the same state, discard the one that has a larger partial path metric and call the remaining path the *survivor*.

Step 4: Extend only the survivor paths to the next interval. Repeat Steps 2 to 4 till the end of the sequence.

Example 6.2 I



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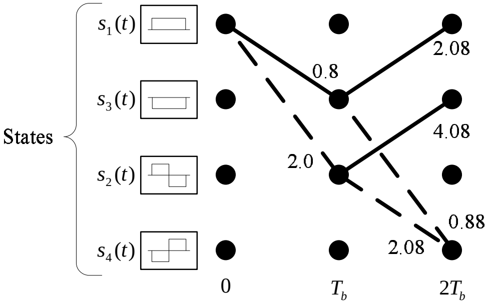
Example 6.2 II



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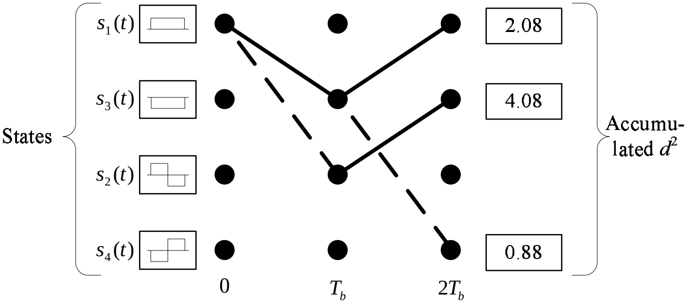
Example 6.2 III



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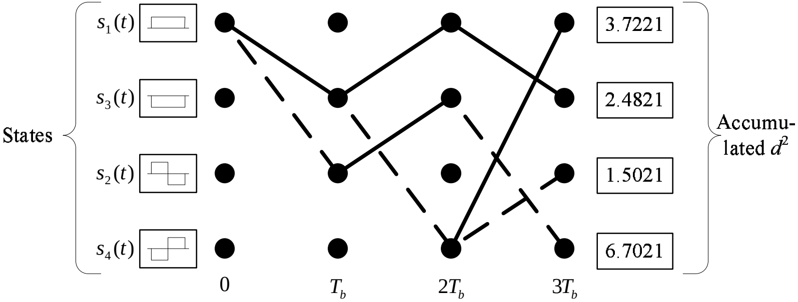
Example 6.2 IV



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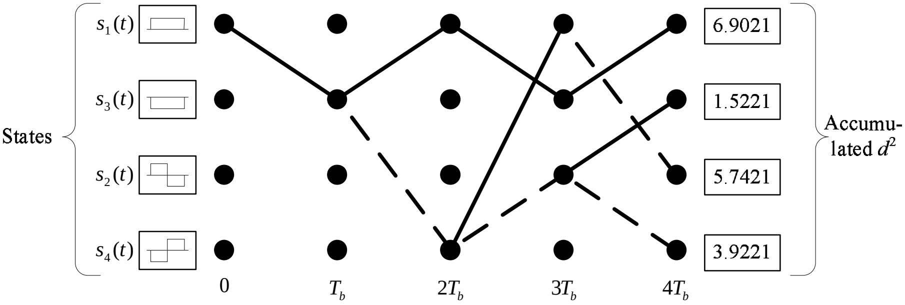
Example 6.2 V



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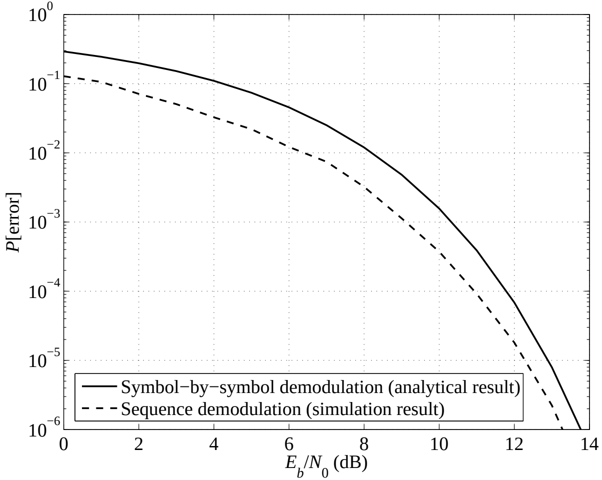
Symbol-by-Symbol vs. Sequence Demodulation



2 dB gain at the error probability of 10^{-2} and 0.5 dB at 10^{-6}.

Spectrum I

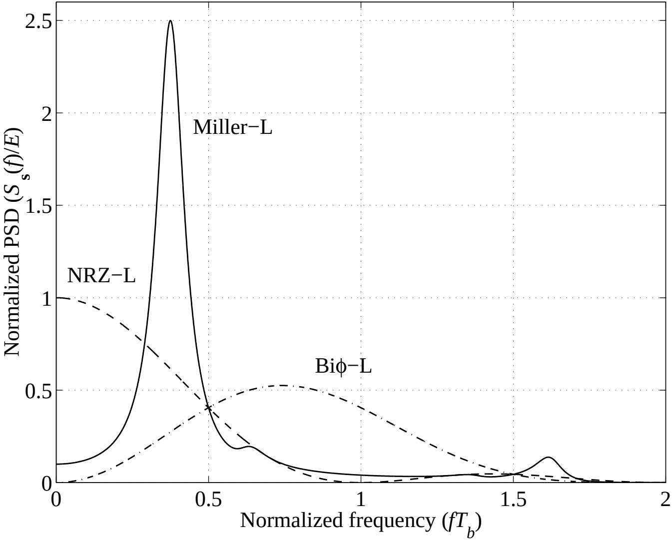
$$\frac{S_{\text{NRZ-L}}(f)}{E} = \frac{1}{T_b}(1-2P)^2\delta(f) + 4P(1-P)\frac{\sin^2(\pi f T_b)}{(\pi f T_b)^2}.$$

$$\begin{aligned}\frac{S_{\text{Bi}\phi}(f)}{E} &= \frac{1}{T_b}(1-2P)^2 \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \left(\frac{2}{n\pi}\right)^2 \delta\left(f - \frac{n}{T_b}\right) \\ &+ 4P(1-P)\frac{\sin^4(\pi f T_b/2)}{(\pi f T_b/2)^2}.\end{aligned}$$

$$\frac{S_{\text{M-L}}(f)}{E} = \frac{1}{2\theta^2(17+8\cos 8\theta)}(23 - 2\cos\theta - 22\cos 2\theta - 12\cos 3\theta$$
$$+5\cos 4\theta + 12\cos 5\theta + 2\cos 6\theta - 8\cos 7\theta + 2\cos 8\theta),$$

where $\theta = \pi f T_b$ and $P_2 = P_1 = 0.5$.

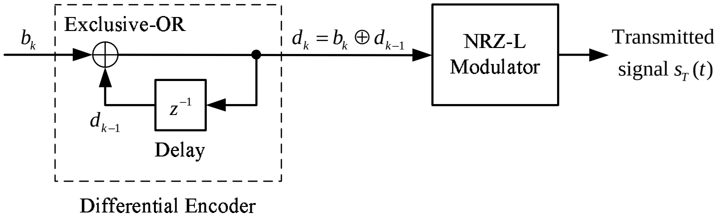
Spectrum II


2.5
2
1.5
1
0.5
0
Normalized PSD ($S_s(f)/E$)
Miller−L
NRZ−L
Biϕ−L
0
0.5
1
1.5
2
Normalized frequency (fT_b)

Differential NRZ-L Modulation

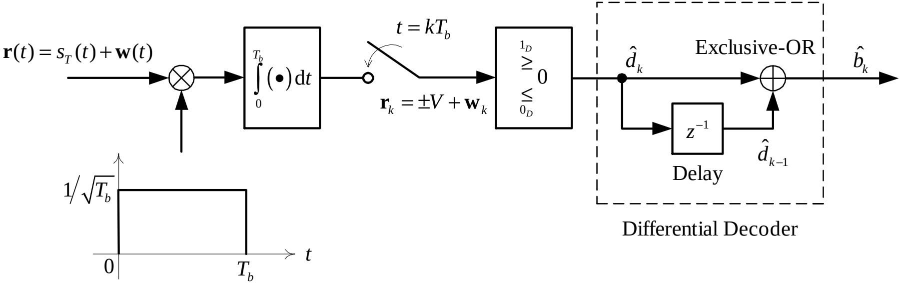
In differential modulation, the signal transmitted in one bit interval is relative to the one transmitted in the previous interval.

- If the present bit is a "1", then transmit a level that is opposite to that of the previous interval.
- If the present bit is a "0", then stay at the same level.



If $b_k = 1$ then $d_k = \overline{d}_{k-1}$, implying a level change and if $b_k = 0$ then $d_k = d_{k-1}$, which means no level change.

Demodulation of Differential NRZ-L



- First determine d_k and call this estimate $\hat{d}_k$.
- To recover b_k, note that $d_k \oplus d_{k-1} = b_k \oplus d_{k-1} \oplus d_{k-1} = b_k$
- The receiver uses $\hat{d}_{k-1}$ instead of d_{k-1}.
- If $\hat{d}_k$ is in error, there will be two errors in the sequence $\{\hat{b}_k\}$.