A First Course in Digital Communications

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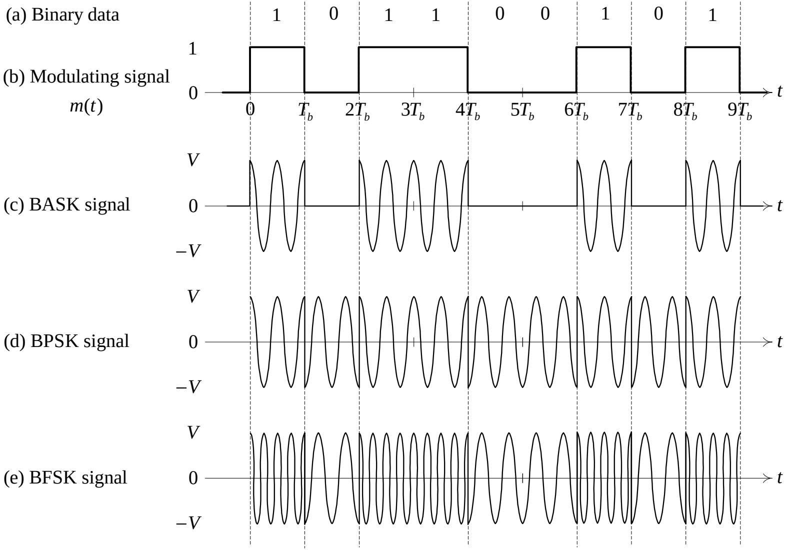
Introduction

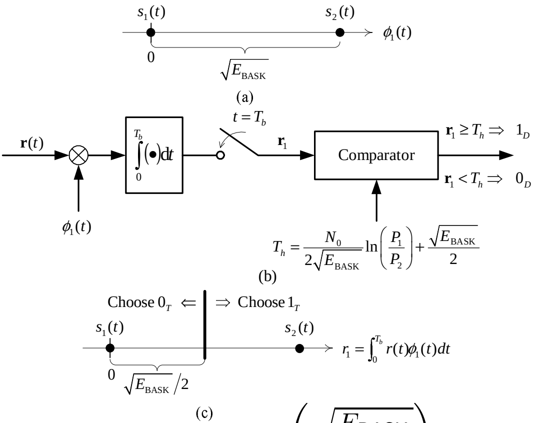
- Baseband transmission is conducted at low frequencies.
- Passband transmission happens in a frequency band toward the high end of the spectrum.
- Satellite communication is in the 6–8 GHz band, while mobile phones systems are in the 800 MHz–2.0 GHz band.
- Bits are encoded as a variation of the *amplitude*, *phase* or *frequency*, or some combination of these parameters of a sinusoidal *carrier*.
- The carrier frequency is much higher than the highest frequency of the modulating signals (or messages).
- Shall consider binary *amplitude-shift keying* (BASK), binary *phase-shift keying* (BPSK) and binary *frequency-shift keying* (BFSK): Error performance, optimum receivers, spectra.
- Extensions to *quadrature phase-shift keying* (QPSK), offset QPSK (OQPSK) and minimum shift keying (MSK).

Examples of Binary Passband Modulated Signals



Binary Amplitude-Shift Keying (BASK)

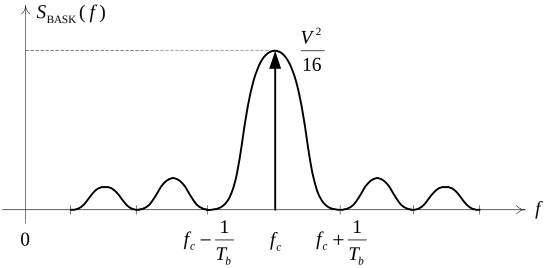
$$\begin{cases} s_1(t) = 0, & \text{“}0_T\text{”} \\ s_2(t) = V\cos(2\pi f_c t), & \text{“}1_T\text{”} \end{cases}, \quad 0 < t \le T_b,\ f_c = n/T_b$$



$$P[\text{error}]_{\text{BASK}} = Q\left(\sqrt{\frac{E_{\text{BASK}}}{2N_0}}\right).$$

PSD of BASK

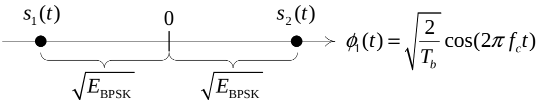
$$S_{\mathsf{BASK}}(f) = \frac{V^2}{16}\left[\delta(f-f_c)+\delta(f+f_c)+ \frac{\sin^2[\pi T_b(f+f_c)]}{\pi^2 T_b(f+f_c)^2}+\frac{\sin^2[\pi T_b(f-f_c)]}{\pi^2 T_b(f-f_c)^2}\right].$$



Approximately 95% of the total transmitted power lies in a band of $3/T_b$ (Hz), centered at f_c.

Binary Phase-Shift Keying (BPSK)

$$\begin{cases} s_1(t) = -V\cos(2\pi f_c t), & \text{if “}0_T\text{”} \\ s_2(t) = +V\cos(2\pi f_c t), & \text{if “}1_T\text{”} \end{cases}, \quad 0 < t \le T_b,$$

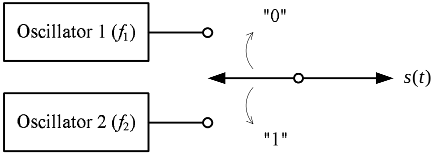


$$P[\text{error}]_{\text{BPSK}} = Q\left(\sqrt{\frac{2E_{\text{BPSK}}}{N_0}}\right).$$

$$S_{\text{BPSK}}(f) = \frac{V^2}{4}\left[\frac{\sin^2[\pi(f-f_c)T_b]}{\pi^2(f-f_c)^2)T_b} + \frac{\sin^2[\pi(f+f_c)T_b]}{\pi^2(f+f_c)^2T_b}\right].$$

Similar to that of BASK, but no impulse functions at $\pm f_c$.

Binary Frequency-Shift Keying (BFSK)



(a)

$$\begin{cases} s_1(t) = V\cos(2\pi f_1 t + \theta_1), & \text{if "}0_T\text{"} \\ s_2(t) = V\cos(2\pi f_2 t + \theta_2), & \text{if "}1_T\text{"} \end{cases}, \quad 0 < t \le T_b.$$

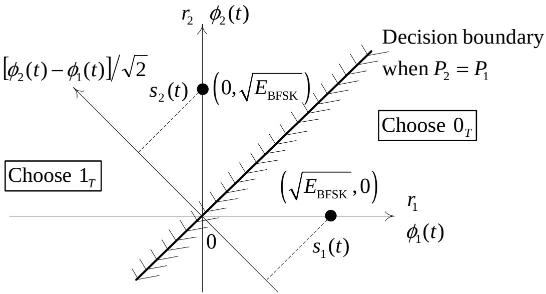
(i) Minimum frequency separation for *coherent* orthogonality ($\theta_1 = \theta_2$):

$$(\Delta f)_{\min}^{[\text{coherent}]} = \frac{1}{2T_b}.$$

(ii) Minimum frequency separation for *noncoherent* orthogonality ($\theta_1 \neq \theta_2$):

$$(\Delta f)_{\min}^{[\text{noncoherent}]} = \frac{1}{T_b}.$$

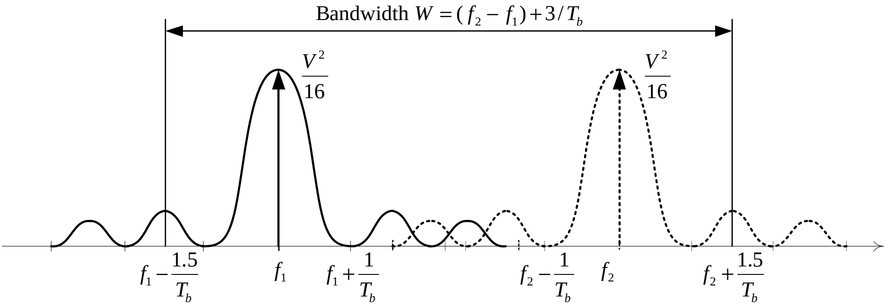
$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_{\mathsf{BFSK}}}}, \quad \phi_2(t) = \frac{s_2(t)}{\sqrt{E_{\mathsf{BFSK}}}}.$$



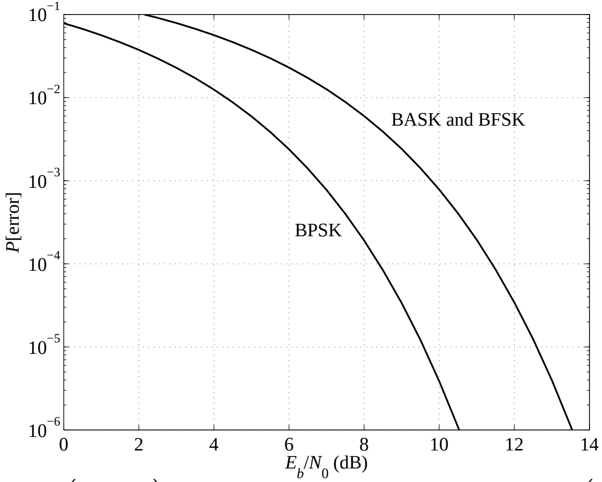
$$P[\mathsf{error}]_{\mathsf{BFSK}} = Q\left(\sqrt{\frac{E_{\mathsf{BFSK}}}{N_0}}\right).$$

PSD of BFSK

$$\begin{aligned} S_{\mathsf{BFSK}}(f) &= \frac{V^2}{16}\left[\delta(f-f_2)+\delta(f+f_2)+\frac{\sin^2[\pi T_b(f+f_2)]}{\pi^2 T_b(f+f_2)^2}+\frac{\sin^2[\pi T_b(f-f_2)]}{\pi^2 T_b(f-f_2)^2}\right] \\ &+ \frac{V^2}{16}\left[\delta(f-f_1)+\delta(f+f_1)+\frac{\sin^2[\pi T_b(f+f_1)]}{\pi^2 T_b(f+f_1)^2}+\frac{\sin^2[\pi T_b(f-f_1)]}{\pi^2 T_b(f-f_1)^2}\right]. \end{aligned}$$



Performance Comparison of BASK, BPSK and BFSK

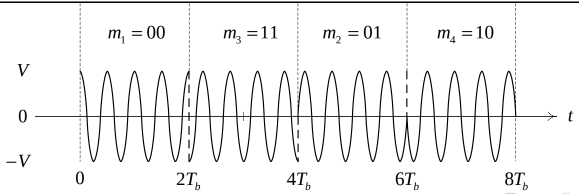


$$P[\text{error}]_{\text{BPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad P[\text{error}]_{\text{BASK}} = P[\text{error}]_{\text{BFSK}} = Q\left(\sqrt{\frac{E_b}{N_0}}\right).$$

Quadrature Phase Shift Keying (QPSK)

- Basic idea behind QPSK: $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are orthogonal over $[0, T_b]$ when $f_c = k/T_b$, k integer $\Rightarrow$ Can transmit two different bits over the same frequency band at the same time.
- The symbol signaling rate (i.e., the *baud rate*) is $r_s = 1/T_s = 1/(2T_b) = r_b/2$ (symbols/sec), i.e., *halved*.

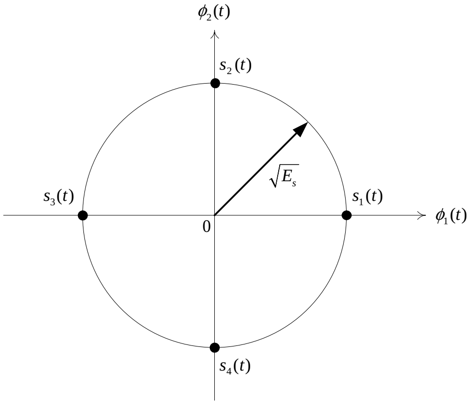
Bit Pattern	Message	Signal Transmitted
00	m_1	$s_1(t) = V\cos(2\pi f_c t), \quad 0 \le t \le T_s = 2T_b$
01	m_2	$s_2(t) = V\sin(2\pi f_c t), \quad 0 \le t \le T_s = 2T_b$
11	m_3	$s_3(t) = -V\cos(2\pi f_c t), \quad 0 \le t \le T_s = 2T_b$
10	m_4	$s_4(t) = -V\sin(2\pi f_c t), \quad 0 \le t \le T_s = 2T_b$



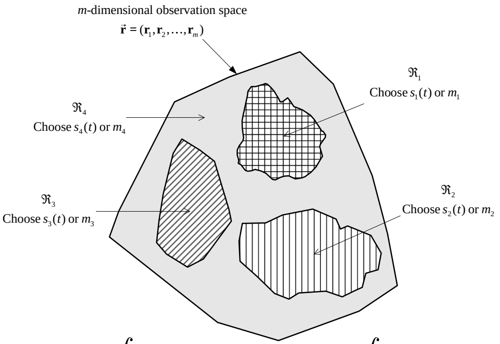
Signal Space Representation of QPSK

$$\int_0^{T_s} s_i^2(t)\mathrm{d}t = \frac{V^2}{2}T_s = V^2 T_b = E_s,$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_s}}, \quad \phi_2(t) = \frac{s_2(t)}{\sqrt{E_s}}.$$



Optimum Receiver for QPSK



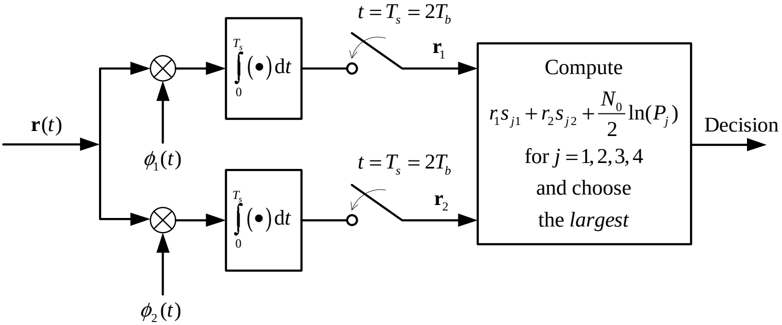
$$P[\text{correct}] = \int_{\Re_1} P_1 f(\vec{r}|s_1(t))\mathrm{d}\vec{r} + \int_{\Re_2} P_2 f(\vec{r}|s_2(t))\mathrm{d}\vec{r} + \int_{\Re_3} P_3 f(\vec{r}|s_3(t))\mathrm{d}\vec{r} + \int_{\Re_4} P_4 f(\vec{r}|s_4(t))\mathrm{d}\vec{r}.$$

Choose $s_i(t)$ if $P_i f(\vec{r}|s_i(t)) > P_j f(\vec{r}|s_j(t)),\ j = 1,2,3,4;\ j \neq i.$

Simplified Decision Rule and Receiver Implementation

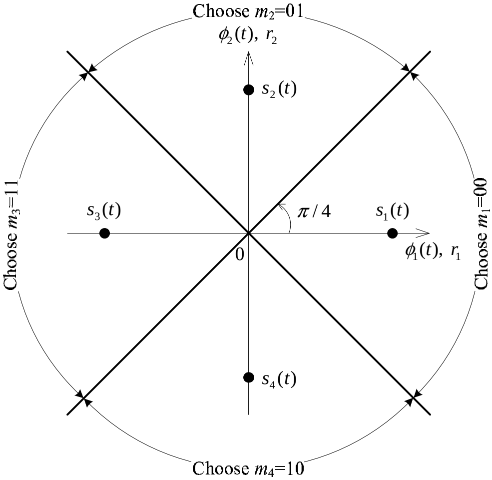
Choose $s_i(t)$ if

$$\frac{N_0}{2}\ln P_i + r_1 s_{i1} + r_2 s_{i2} > \frac{N_0}{2}\ln P_j + r_1 s_{j1} + r_2 s_{j2}$$
$$j = 1, 2, 3, 4;\ \ j \neq i.$$

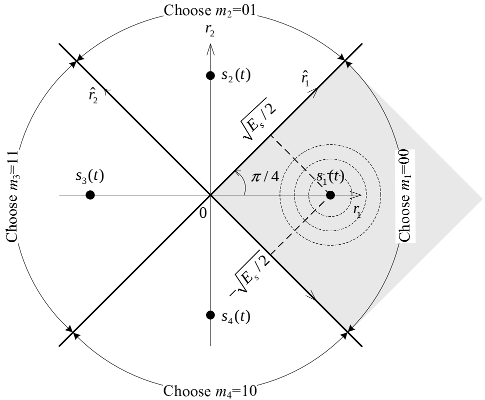


Minimum-Distance Receiver

Choose $s_i(t)$ if $(r_1 - s_{i1})^2 + (r_2 - s_{i2})^2$ is *the smallest*

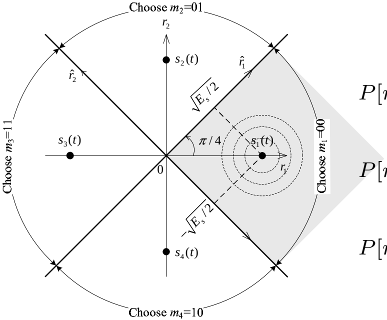


Symbol (Message) Error Probability of QPSK



$$P[\text{error}] = P[\text{error}|s_i(t)] = 1 - P[\text{correct}|s_i(t)] = 1 - \left[1 - Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right]^2.$$

Bit Error Probability of QPSK



$$P[m_2|m_1] = Q\left(\sqrt{\frac{E_s}{N_0}}\right)\left[1-Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right],$$

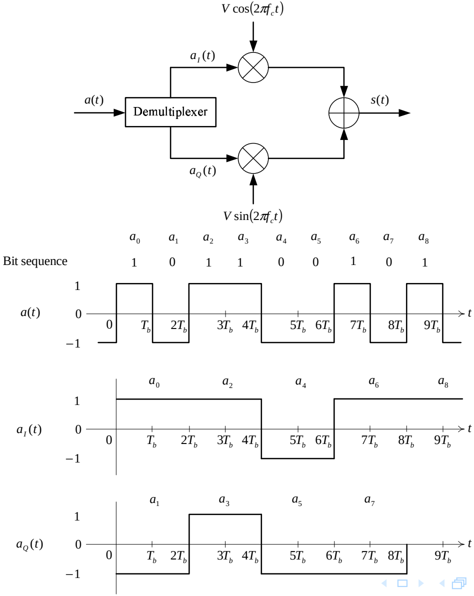
$$P[m_3|m_1] = Q^2\left(\sqrt{\frac{E_s}{N_0}}\right),$$

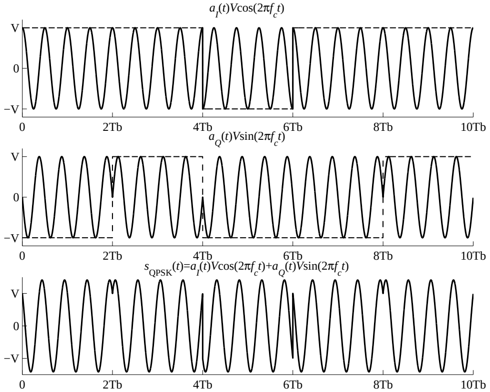
$$P[m_4|m_1] = Q\left(\sqrt{\frac{E_s}{N_0}}\right)\left[1-Q\left(\sqrt{\frac{E_s}{N_0}}\right)\right].$$

$$P[\text{bit error}] = 0.5P[m_2|m_1] + 0.5P[m_4|m_1] + 1.0P[m_3|m_1]$$
$$= Q\left(\sqrt{\frac{E_s}{N_0}}\right).$$

Gray mapping: *Nearest neighbors* are mapped to the bit pairs that differ in only one bit.

An Alternative Representation of QPSK



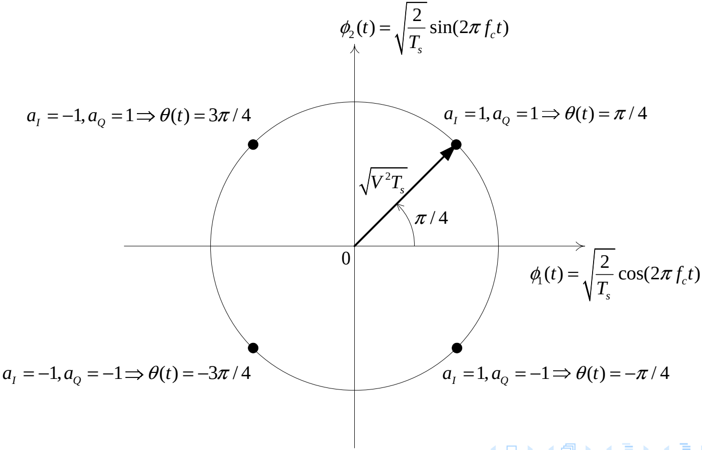


$$s(t) = a_I(t)V\cos(2\pi f_c t) + a_Q(t)V\sin(2\pi f_c t)$$
$$= \sqrt{a_I^2(t) + a_Q^2(t)}V\cos\left(2\pi f_c t - \tan^{-1}\left(\frac{a_Q(t)}{a_I(t)}\right)\right) = \sqrt{2}V\cos[2\pi f_c t - \theta(t)].$$

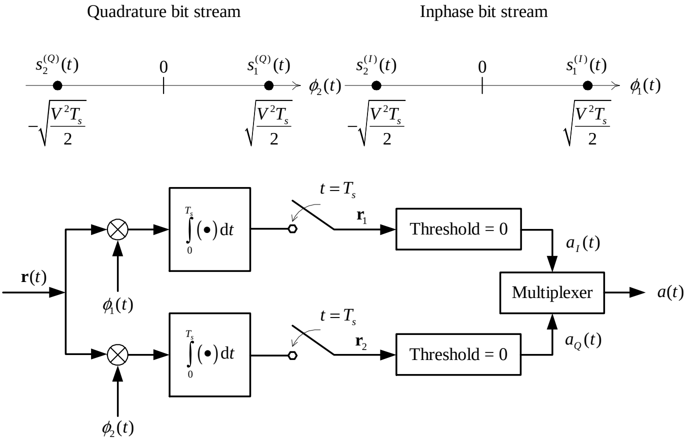
$$\theta(t) = \begin{cases} \pi/4, & \text{if } a_I = +1, a_Q = +1 \text{ (bits are 11)} \\ -\pi/4, & \text{if } a_I = +1, a_Q = -1 \text{ (bits are 10)} \\ 3\pi/4, & \text{if } a_I = -1, a_Q = +1 \text{ (bits are 01)} \\ -3\pi/4, & \text{if } a_I = -1, a_Q = -1 \text{ (bits are 00)} \end{cases}.$$

Signal Space Representation of QPSK

$$\begin{cases} \phi_1(t) = \dfrac{V\cos(2\pi f_c t)}{\sqrt{V^2 T_b}} \\ \phi_2(t) = \dfrac{V\sin(2\pi f_c t)}{\sqrt{V^2 T_b}} \end{cases}, \quad 0 < t < T_s = 2T_b,$$



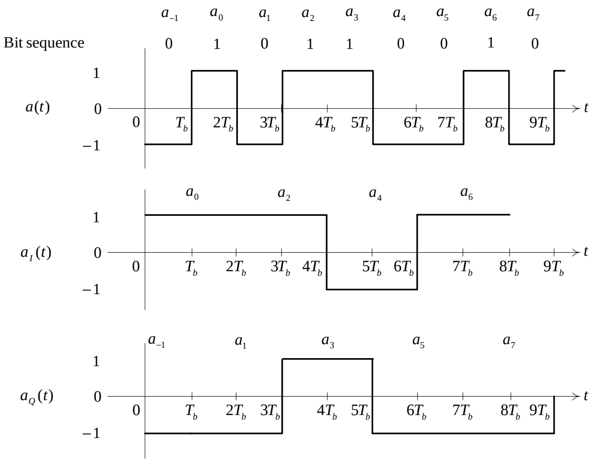
Receiver Implementation of QPSK



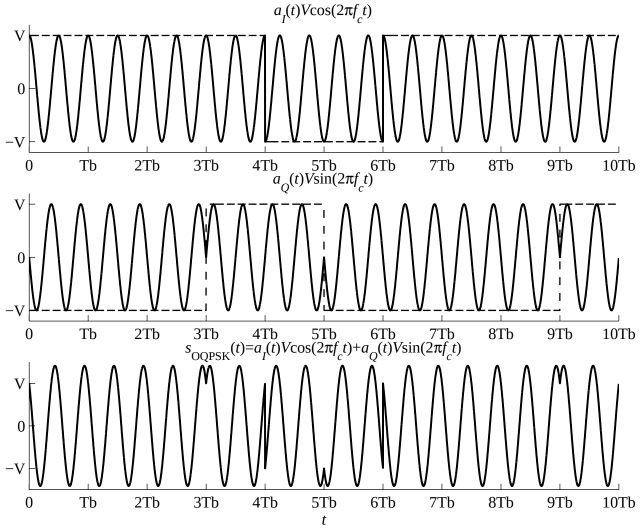
$$P[\text{bit error}] = Q\left(\sqrt{\frac{V^2T_s}{N_0}}\right) = \cdots = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

Offset Quadrature Phase Shift Keying (OQPSK)

- In OQPSK the $a_I(t)$ and $a_Q(t)$ bit streams are offset by one bit interval T_b.

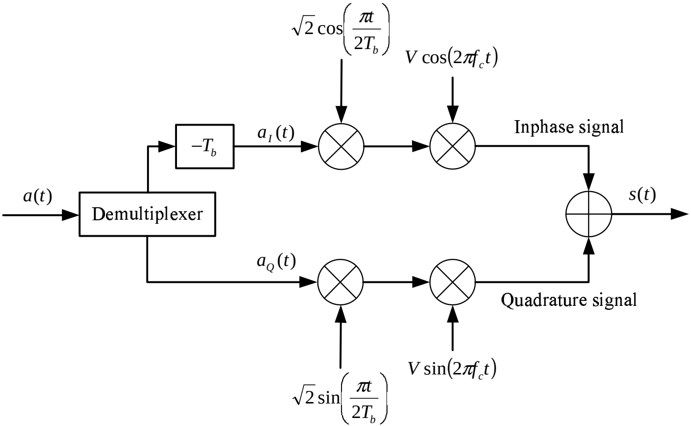


Example of an OQPSK Signal

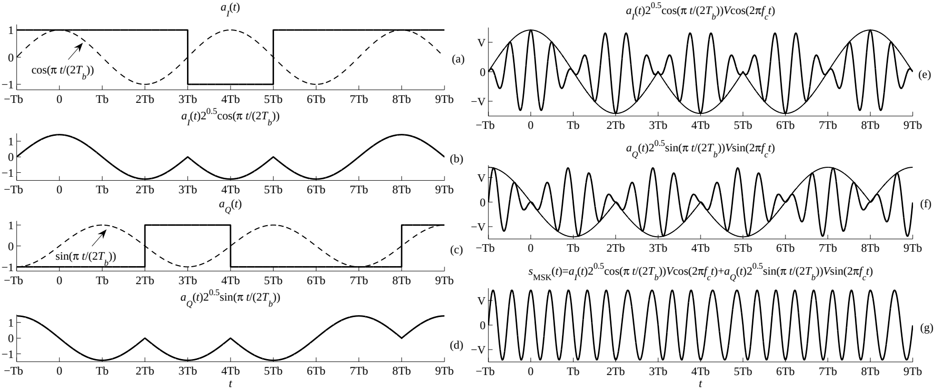


Minimum Shift Keying (MSK)

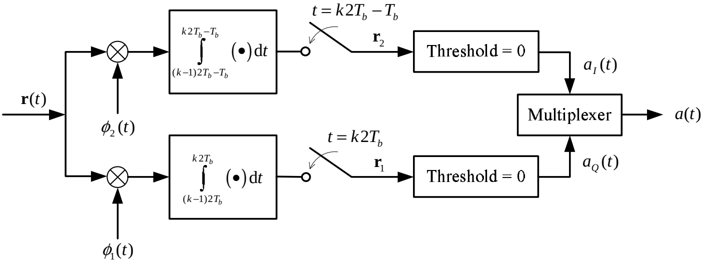
- Both QPSK and OQPSK signals have sudden jumps.
- MSK eliminates the jumps altogether by applying weighting functions to the carriers $V\cos(2\pi f_c t)$ and $V\sin(2\pi f_c t)$.



Generation of an MSK Signal



Block Diagram of MSK Receiver



$$\phi_1(t) = \left[\sqrt{2}\sin\left(\frac{\pi t}{2T_b}\right) V\sin(2\pi f_c t)\right] \Big/ \sqrt{V^2 T_b},$$

$$\phi_2(t) = \left[\sqrt{2}\cos\left(\frac{\pi t}{2T_b}\right) V\cos(2\pi f_c t)\right] \Big/ \sqrt{V^2 T_b}.$$

$$P[\text{bit error}] = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad E_b = V^2 T_b \text{ is the energy per bit.}$$

A Mathematical Description of MSK Signals

$$\begin{aligned} s(t) &= a_I(t)\sqrt{2}\cos\left(\frac{\pi t}{2T_b}\right)V\cos(2\pi f_c t) + a_Q(t)\sqrt{2}\sin\left(\frac{\pi t}{2T_b}\right)V\sin(2\pi f_c t) \\ &= A\cos(2\pi f_c t - \theta). \end{aligned}$$

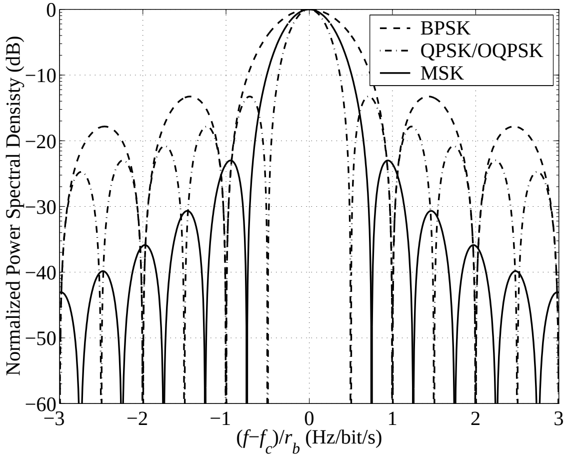
$$A = \left[a_I^2(t)2V^2\cos^2\left(\frac{\pi t}{2T_b}\right) + a_Q^2(t)2V^2\sin^2\left(\frac{\pi t}{2T_b}\right)\right]^{\frac{1}{2}} = \sqrt{2}V$$

$$\theta = \tan^{-1}\left\{\frac{a_Q(t)\sin\left(\frac{\pi t}{2T_b}\right)}{a_I(t)\cos\left(\frac{\pi t}{2T_b}\right)}\right\} = \tan^{-1}\left\{\pm\tan\left(\frac{\pi t}{2T_b}\right)\right\} = \pm\frac{\pi t}{2T_b}.$$

$$\Rightarrow s(t) = \sqrt{2}V\cos\left[2\pi\left(f_c \pm \frac{1}{4T_b}\right)t\right].$$

An MSK signal is of either frequency $f_2 = f_c + \frac{1}{4T_b}$ or $f_1 = f_c - \frac{1}{4T_b}$ $\Rightarrow$ can be viewed as *frequency-shift keying* signal with continuous phase.

Power Spectral Density



$$S_{\mathsf{MSK}}(f) = K\left\{\left[\frac{\cos[2\pi(f-f_c)T_b]}{4\pi^2(f-f_c)^2-\pi^2/(4T_b^2)}\right]^2+\left[\frac{\cos[2\pi(f+f_c)T_b]}{4\pi^2(f+f_c)^2-\pi^2/(4T_b^2)}\right]^2\right\}.$$

Modulation in 2G Cellular Wireless Systems

	GSM/DCS-1800	IS-54/136	PDC	1S-95
Region	Europe	North America	Japan	North America
Frequency band (MHz)	900/1800/1900	800/1900	700/1500	800/1900
Multiple access	F/TDMA	F/TDMA	F/TDMA	F/CDMA
Carrier spacing (kHz)	200	30	25	1250
Modulation	GMSK	OQPSK	OQPSK	BPSK/QPSK
Speech coding (kb/s)	VSELP (HR-5.6) RPE-LTP (FR-13) ACELP (EFR-12.2)	VSELP (FR-7.95) ACELP (EFR-7.4)	PSI-CELP (HR-3.45) VSELP (FR-6.7)	QCELP (8, 4, 2, 1) RCELP (EVRC)
Frame size (ms)	4.6	40	20	20
Channel coding (convolution code)	Rate 1/2	Rate 1/2	Rate 1/2	Rate 1/2 or 1/3

HR: half-rate codec; FR: full-rate codec; EFR: enhanced full-rate codec; EVRC: enhanced variable rate codec

An adaptive multitate (AMR) codec for GSM is currently being standardized by ETSI

Table 1. *Air interface characteristics of 2G systems.*

Modulation in 3G CDMA-Based Cellular Systems

Proposal	UTRA	cdma2000	WCDMA/NA	WIMS W-CDMA	W-CDMA	TD-SCDMA	CDMA II	CDMA I
Multiple-access	FDD: DS-CDMA TDD: T/CDMA	FDD: DS-CDMA TDD: T/CDMA	FDD: DS-CDMA TDD: T/CDMA	FDD: DS-CDMA TDD: DS-W-CDMA(FL) DS-S-TDMA (RL)	FDD: DS-CDMA TDD: T/CDMA	TDMA/CDMA	DS-CDMA	DS-CDMA
Duplex scheme	FDD/TDD	FDD/TDD	FDD/TDD	W-CDMA FDD mode: FDD S-TDMA TDD mode: TDD	FDD/TDD	TDD	FDD	FDD
Chip rate (Mc/s)	FDD: 4.096/ 8.192/16.384 TDD: 4.096	1.2288xN Mcps (NX)	FDD: 4.096/8.192/ 16.384 TDD: 4.096	4.096/8.192/ 16.384	1.024/4 096/ 8.192/16 384	1.1136	1.024/4.096/ 8.192/16.384	0.9216/3.6864/ 14.7456
Frame length	10 ms	20/5 ms	10 ms	10 ms	10 ms	5 ms	10 ms	10 ms
Channel coding	Convolutional coding (rate 1/2, 1/3, $K = 9$); optional outer RS coding (rate TBD)	Convolutional coding (R=1/2, 1/3, 1/4, $K = 9$); Turbo code of R=1/2, 1/3, 1/4 and $K = 4$ (preferred for date transmission over 14.4 kb/s on supplemental channel)	Convolutional coding (rate 1/2, 1/3, $K = 9$); optional outer RS coding ($R = 4/5$)	Convolutional coding (FL: $R = 1/2$, $K = 7$, RL: $R = 1/3$, $K = 9$)	Convolutional coding ($R = 1/2, 1/3$, $K = 9$); Turbo code of $R = 1/3$ $K = 3$ (data transmission over 32 kb/s)	Convolutional coding ($R = 3/4$, $\boldsymbol{K} = 9$); optional outer RS code; Turbo code of $K = 4$, $R = 1/2$ (preferred for data rate greater than 19.2 kb/s NRT service)	Convolutional coding ($R = 1/2, 1/3, 1/4, 1/6$, $K = 9$), selectable FEC for low rate data; Turbo code of $R = 1/3$ and $K = 3$ for high rate data and packet data	Convolutional coding $R = 1/2, 1/3, 1/4, 1/6$); optional outer (47, 41) RS code
Interleaving	Inter/intraframe	Intraframe	Inter/intraframe	Block interleaving (no details given)	Multistage intra or interframe	Interframe,	Intraframe	Intraframe
Data modulation	FDD: FL: QPSK, RL: Dual-channel-QPSK; TDD: QPSK (RL&FL)	QPSK (FL) BPSK (RL)	FDD: FL: QPSK, RL: Dual-channel QPSK; TDD: QPSK (RL&FL)	QPSK	FDD: FL: QPSK, RL: Dual-channel QPSK TDD: QPSK	DQPSK, and 16QAM for high data rate	QPSK (FL) BPSK (RL)	FL: QPSK RL: BPSK