Introduction

- There are benefits to be gained when $M$-ary ($M = 4$) signaling methods are used rather than straightforward binary signaling.
- In general, $M$-ary communication is used when one needs to design a communication system that is bandwidth efficient.
- Unlike QPSK and its variations, the gain in bandwidth is accomplished at the expense of error performance.
- To use $M$-ary modulation, the bit stream is blocked into groups of $\lambda$ bits $\Rightarrow$ the number of bit patterns is $M = 2^\lambda$.
- The symbol transmission rate is $r_s = 1/T_s = 1/(\lambda T_b) = r_b/\lambda$ symbols/sec $\Rightarrow$ there is a bandwidth saving of $1/\lambda$ compared to binary modulation.
- Shall consider $M$-ary ASK, PSK, QAM (quadrature amplitude modulation) and FSK.
Optimum Receiver for $M$-ary Signaling

- $w(t)$ is zero-mean white Gaussian noise with power spectral density of $\frac{N_0}{2}$ (watts/Hz).
- Receiver needs to make the decision on the transmitted signal based on the received signal $r(t) = s_i(t) + w(t)$.
- The determination of the optimum receiver (with minimum error) proceeds in a manner analogous to that for the binary case.
Represent $M$ signals by an orthonormal basis set, $\{\phi_n(t)\}_{n=1}^{N}, N \leq M$:

$$s_i(t) = s_{i1}\phi_1(t) + s_{i2}\phi_2(t) + \cdots + s_{iN}\phi_N(t),$$

$$s_{ik} = \int_0^{T_s} s_i(t)\phi_k(t)dt.$$

Expand the received signal $r(t)$ into the series

$$r(t) = s_i(t) + w(t)$$

$$= r_1\phi_1(t) + r_2\phi_2(t) + \cdots + r_N\phi_N(t) + r_{N+1}\phi_{N+1}(t) + \cdots$$

For $k > N$, the coefficients $r_k$ can be discarded.

Need to partition the $N$-dimensional space formed by $\vec{r} = (r_1, r_2, \ldots, r_N)$ into $M$ regions so that the message error probability is minimized.
Chapter 8: \(M\)-ary Signaling Techniques

The optimum receiver is also the \textit{minimum-distance receiver}:

Choose \(m_i\) if

\[
\sum_{k=1}^{N} (r_k - s_{ik})^2 < \sum_{k=1}^{N} (r_k - s_{jk})^2; \\
j = 1, 2, \ldots, M; \ j \neq i.
\]
\( M \)-ary Coherent Amplitude-Shift Keying (\( M \)-ASK)

\[
s_i(t) = V_i \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_s
\]

\[
= [(i - 1)\Delta] \phi_1(t), \quad \phi_1(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_s,
\]

\( i = 1, 2, \ldots, M. \)

---

**Diagram:**

- \( s_1(t) \) to \( s_M(t) \) with time intervals from 0 to \( (M - 1)\Delta \).
- \( \phi_1(t) \) as a function of time.

**Signal Flow:**

- \( s_i(t) \) and \( \phi_1(t) \) are inputs to the decision device.
- \( r(t) \) and \( w(t) \) are inputs to the integrator.
- \( \int_{(k-1)T_s}^{kT_s} (\cdot) \, dt \) is the integrator with output \( r_1(t) \).
- \( t = kT_s \) is the decision threshold.
- \( \hat{m}_i \) is the decision output.

**Noise:**

- \( WGN, \text{ strength } \frac{N_0}{2} \text{ watts/Hz} \).
Choose \[ s_k(t), \quad \text{if} \quad (k - \frac{3}{2}) \Delta < r_1 < (k - \frac{1}{2}) \Delta, \quad k = 2, 3, \ldots, M - 1 \]

Choose \[ s_1(t), \quad \text{if} \quad r_1 < \frac{\Delta}{2} \]

Choose \[ s_M(t), \quad \text{if} \quad r_1 > \left( M - \frac{3}{2} \right) \Delta \]

Choose \[ f(r_1|s_k(t)) \]

Choose \[ s_1(t) \]

Choose \[ s_k(t) \]

Choose \[ s_M(t) \]
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Error Performance of $M$-ASK

$$P[\text{error}] = \sum_{i=1}^{M} P[s_i(t)] P[\text{error} \mid s_i(t)].$$

$$P[\text{error} \mid s_i(t)] = 2Q \left( \Delta / \sqrt{2N_0} \right), \quad i = 2, 3, \ldots, M - 1.$$  

$$P[\text{error} \mid s_i(t)] = Q \left( \Delta / \sqrt{2N_0} \right), \quad i = 1, M.$$  

$$P[\text{error}] = \frac{2(M - 1)}{M} Q \left( \Delta / \sqrt{2N_0} \right).$$
Modified $M$-ASK Constellation

The maximum and average transmitted energies can be reduced, without any sacrifice in error probability, by changing the signal set to one which includes the negative version of each signal.

$$s_i(t) = (2i - 1 - M) \frac{\Delta}{2} \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t), \ 0 \leq t \leq T_s, \ i = 1, 2, \ldots, M.$$
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Probability of Symbol Error for $M$-ASK

\[ P[\text{error}] = \frac{2(M - 1)}{M} Q \left( \sqrt{\frac{6E_s}{(M^2 - 1)N_0}} \right) = \frac{2(M - 1)}{M} Q \left( \sqrt{\frac{6 \log_2 M E_b}{M^2 - 1 N_0}} \right). \]

\[ P[\text{bit error}] = \frac{1}{\lambda} P[\text{symbol error}] = \frac{2(M - 1)}{M \log_2 M} Q \left( \sqrt{\frac{6 \log_2 M E_b}{M^2 - 1 N_0}} \right) \text{ (with Gray mapping)} \]

\[ W \text{ is obtained by using the } WT_s = 1 \text{ rule-of-thumb. Here } 1/T_b \text{ is the bit rate (bits/s)}. \]
Example of 2-ASK (BPSK) and 4-ASK Signals

Baseband information signal

BPSK Signalling

4-ASK Signalling
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**M-ary Phase-Shift Keying (M-PSK)**

\[
s_i(t) = V \cos \left[ 2\pi f_c t - \frac{(i - 1)2\pi}{M} \right], \quad 0 \leq t \leq T_s, \quad i = 1, 2, \ldots, M; \quad f_c = k/T_s, \quad k \text{ integer}; \quad E_s = V^2 T_s/2 \text{ joules}
\]

\[
s_i(t) = V \cos \left( \frac{(i - 1)2\pi}{M} \right) \cos(2\pi f_c t) + V \sin \left( \frac{(i - 1)2\pi}{M} \right) \sin(2\pi f_c t).
\]

\[
\phi_1(t) = \frac{V \cos(2\pi f_c t)}{\sqrt{E_s}}, \quad \phi_2(t) = \frac{V \sin(2\pi f_c t)}{\sqrt{E_s}}.
\]

\[
s_{i1} = \sqrt{E_s} \cos \left( \frac{(i - 1)2\pi}{M} \right), \quad s_{i2} = \sqrt{E_s} \sin \left( \frac{(i - 1)2\pi}{M} \right).
\]

The signals lie on a circle of radius \( \sqrt{E_s} \), and are spaced every \( 2\pi/M \) radians around the circle.
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Signal Space Plot of 8-PSK

\[ s_i(t) = V \cos \left[ 2\pi f_c t - \frac{(i - 1)2\pi}{M} \right], \quad 0 \leq t \leq T_s, \]

\( i = 1, 2, \ldots, M; \quad f_c = k/T_s, \quad k \text{ integer}; \quad E_s = V^2 T_s / 2 \text{ joules} \)
Signal Space Plot of General $M$-PSK

$$s_i(t) = V \cos \left[ 2\pi f_c t - \frac{(i - 1)2\pi}{M} \right], \quad 0 \leq t \leq T_s,$$

$i = 1, 2, \ldots, M$; $f_c = k/T_s$, $k$ integer; $E_s = V^2 T_s/2$ joules
Chapter 8: \( M \)-ary Signaling Techniques

Optimum Receiver for \( M \)-PSK

\[
\phi_1(t) \quad \phi_2(t)
\]

Compute
\[
(r_i - s_{i1})^2 + (r_2 - s_{i2})^2
\]
for \( i = 1, 2, \ldots, M \)
and choose the smallest

\[
P[\text{error}] = P[\text{error} \mid s_1(t)]
\]
\[
= 1 - \int \int_{r_1, r_2 \in \text{Region 1}} f(r_1, r_2 \mid s_1(t)) \, dr_1 \, dr_2.
\]
Lower Bound of \( P[\text{error}] \) of \( M \)-PSK

\[
P[\text{error}|s_1(t)] > P[r_1, r_2 \text{ fall in } \mathcal{R}_1|s_1(t)], \text{ or}
\]

\[
P[\text{error}|s_1(t)] > Q\left\{ \sin \left( \frac{\pi}{M} \right) \sqrt{2E_s/N_0} \right\}.
\]
Upper Bound of $P[\text{error}]$ of $M$-PSK

\[
P[\text{error}] < P[r_1, r_2 \text{ fall in } \mathcal{R}_1|s_1(t)] + P[r_1, r_2 \text{ fall in } \mathcal{R}_2|s_1(t)], \text{ or}
\]
\[
P[\text{error}] < 2Q\left(\sin\left(\frac{\pi}{M}\right) \sqrt{\frac{2E_s}{N_0}}\right),
\]
With a Gray mapping, the bit error probability is approximated as:

\[
P[\text{bit error}]_{M-\text{PSK}} \approx \frac{1}{\log_2 M} Q \left( \sqrt{\lambda \sin^2 \left( \frac{\pi}{M} \frac{2E_b}{N_0} \right)} \right).
\]
Comparison of BPSK and $M$-PSK

\[ P[\text{error}]_{M-\text{PSK}} \simeq Q \left( \sqrt{\lambda \sin^2 \left( \frac{\pi}{M} \right)} \frac{2E_b}{N_0} \right), \quad \text{where } E_s = \lambda E_b. \]

\[ P[\text{error}]_{\text{BPSK}} = Q(\sqrt{2E_b/N_0}). \]

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$M$</th>
<th>$M$-ary BW/Binary BW</th>
<th>$\lambda \sin^2(\pi/M)$</th>
<th>$M$-ary Energy/Binary Energy</th>
</tr>
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<tbody>
<tr>
<td>3</td>
<td>8</td>
<td>1/3</td>
<td>0.44</td>
<td>3.6 dB</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>1/4</td>
<td>0.15</td>
<td>8.2 dB</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>1/5</td>
<td>0.05</td>
<td>13.0 dB</td>
</tr>
<tr>
<td>6</td>
<td>64</td>
<td>1/6</td>
<td>0.0144</td>
<td>17.0 dB</td>
</tr>
</tbody>
</table>
**M-ary Quadrature Amplitude Modulation (M-QAM)**

- M-QAM constellations are two-dimensional and they involve inphase (I) and quadrature (Q) carriers:
  
  \[
  \phi_I(t) = \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t), \quad 0 \leq t \leq T_s, \\
  \phi_Q(t) = \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t), \quad 0 \leq t \leq T_s, 
  \]

- The \(i\)th transmitted M-QAM signal is:
  
  \[
  s_i(t) = V_{I,i} \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t) + V_{Q,i} \sqrt{\frac{2}{T_s}} \sin(2\pi f_c t), \quad 0 \leq t \leq T_s \\
  i = 1, 2, \ldots, M \\
  = \sqrt{E_i} \sqrt{\frac{2}{T_s}} \cos(2\pi f_c t - \theta_i)
  \]

\(V_{I,i}\) and \(V_{Q,i}\) are the information-bearing discrete amplitudes of the two quadrature carriers, \(E_i = V_{I,i}^2 + V_{Q,i}^2\) and \(\theta_i = \tan^{-1}(V_{Q,i}/V_{I,i})\).
Chapter 8: M-ary Signaling Techniques

$M = 4$

$M = 8$

Rectangle

Triangle
Chapter 8: $M$-ary Signaling Techniques

- Hexagon
- Triangle
- Hexagon

$16 = M$

- Rectangle
- Triangle
- Circle

$M = 16$
A Simple Comparison of $M$-QAM Constellations

With the same *minimum* distance of all the constellations, a more efficient signal constellation is the one that has smaller average transmitted energy.

$E_s$ for the rectangular, triangular, (1,7) and (4,4) constellations are found to be $1.50\Delta^2$, $1.125\Delta^2$, $1.162\Delta^2$ and $1.183\Delta^2$, respectively.
The signal components take value from the set of discrete values
\[ \left\{ \left(2i - 1 - M\right)\Delta/2 \right\}, \quad i = 1, 2, \ldots, \frac{M}{2}. \]
Modulation of Rectangular $M$-QAM

- Each group of $\lambda = \log_2 M$ bits can be divided into $\lambda_I$ inphase bits and $\lambda_Q$ quadrature bits, where $\lambda_I + \lambda_Q = \lambda$.

- Inphase bits and quadrature bits modulate the inphase and quadrature carriers independently.

![Diagram of M-QAM modulation](attachment:image.png)
Demodulation of Rectangular $M$-QAM

Due to the orthogonality of the inphase and quadrature signals, inphase and quadrature bits can be *independently* detected at the receiver.

The most practical rectangular QAM constellation is one which $\lambda_I = \lambda_Q = \lambda/2$, i.e., $M$ is a perfect square and the rectangle is a square.
Chapter 8: M-ary Signaling Techniques

Symbol Error Probability of \( M \)-QAM

- For square constellations:

\[
P[\text{error}] = 1 - P[\text{correct}] = 1 - \left(1 - P_{\sqrt{M}[\text{error}]\sqrt{M}}\right)^2,
\]

\[
P_{\sqrt{M}[\text{error}]} = 2 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left(\sqrt{\frac{3E_s}{(M - 1)N_0}}\right),
\]

where \( E_s/N_0 \) is the average SNR per symbol.

- For general rectangular constellations:

\[
P[\text{error}] \leq 1 - \left[1 - 2Q \left(\sqrt{\frac{3E_s}{(M - 1)N_0}}\right)\right]^2
\]

\[
\leq 4Q \left(\sqrt{\frac{3\lambda E_b}{(M - 1)N_0}}\right)
\]

where \( E_b/N_0 \) is the average SNR per bit.
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$M=2$, $M=4$, $M=16$, $M=64$, $M=256$

Exact performance

Upper bound

$P_{\text{symbol error}}$ vs. $E_b/N_0$ (dB)
For $M$-PSK, approximate $P[\text{error}] \approx Q\left(\sqrt{\frac{2E_s}{N_0}} \sin \frac{\pi}{M}\right)$.

For $M$-QAM, use the upper bound $4Q\left(\sqrt{\frac{3\lambda E_b}{(M-1)N_0}}\right)$.

Comparing the arguments of $Q(\cdot)$ for the two modulations gives:

$$\kappa_M = \frac{3/(M - 1)}{2 \sin^2(\pi/M)}.$$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$10 \log_{10} \kappa_M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>1.65 dB</td>
</tr>
<tr>
<td>16</td>
<td>4.20 dB</td>
</tr>
<tr>
<td>32</td>
<td>7.02 dB</td>
</tr>
<tr>
<td>64</td>
<td>9.95 dB</td>
</tr>
<tr>
<td>256</td>
<td>15.92 dB</td>
</tr>
<tr>
<td>1024</td>
<td>21.93 dB</td>
</tr>
</tbody>
</table>
Performance Comparison of $M$-ASK, $M$-PSK, $M$-QAM

![Graph showing performance comparison of $M$-ASK, $M$-PSK, and $M$-QAM for different $M$ values.](image-url)

- $M$-ASK
- $M$-PSK
- $M$-QAM

$M=4$, 8, 16, 32

Symbols per bit error rate ($P_{[\text{symbol error}]}$) vs. $E_b/N_0$ (dB)
\( m \)-ary Coherent Frequency-Shift Keying (\( m \)-FSK)

\[
s_i(t) = \begin{cases} 
V \cos(2\pi f_i t), & 0 \leq t \leq T_s \\
0, & \text{elsewhere}
\end{cases}, \quad i = 1, 2, \ldots, M,
\]

where \( f_i \) are chosen to have orthogonal signals over \([0, T_s]\).

\[
f_i = \begin{cases} 
(k \pm i) \left( \frac{1}{2T_s} \right), & (\text{coherently orthogonal}) \\
(k \pm i) \left( \frac{1}{T_s} \right), & (\text{noncoherently orthogonal})
\end{cases}, \quad i = 0, 1, 2, \ldots
\]
Minimum-Distance Receiver of $M$-FSK

Choose $m_i$ if

$$\sum_{k=1}^{M} (r_k - s_{ik})^2 < \sum_{k=1}^{M} (r_k - s_{jk})^2 \quad \Rightarrow \quad j = 1, 2, \ldots, M; \ j \neq i,$$

Choose $m_i$ if

$$r_i > r_j, \quad j = 1, 2, \ldots, M; \ j \neq i.$$
Symbol Error Probability of $M$-FSK

\[ P[\text{error}] = P[\text{error}|s_1(t)] = 1 - P[\text{correct}|s_1(t)]. \]

\[ P[\text{correct}|s_1(t)] = P[(r_2 < r_1) \text{ and } \cdots \text{ and } (r_M < r_1)|s_1(t) \text{ sent}]. \]

\[ = \int_{r_1=-\infty}^{\infty} P[(r_2 < r_1) \text{ and } \cdots \text{ and } (r_M < r_1)|\{r_1 = r_1, s_1(t)\}] f(r_1|s_1(t))dr. \]

\[ P[(r_2 < r_1) \text{ and } \cdots \text{ and } (r_M < r_1)|\{r_1 = r_1, s_1(t)\}] = \prod_{j=2}^{M} P[(r_j < r_1)|\{r_1 = r_1, s_1(t)\}] \]

\[ P[r_j < r_1|\{r_1 = r_1, s_1(t)\}] = \int_{-\infty}^{r_1} \frac{1}{\sqrt{\pi N_0}} \exp \left\{ -\frac{\lambda^2}{N_0} \right\} d\lambda. \]

\[ P[\text{correct}] = \int_{r_1=-\infty}^{\infty} \left[ \int_{\lambda=-\infty}^{r_1} \frac{1}{\sqrt{\pi N_0}} \exp \left\{ -\frac{\lambda^2}{N_0} \right\} d\lambda \right]^{M-1} \times \frac{1}{\sqrt{\pi N_0}} \exp \left\{ -\frac{(r_1 - \sqrt{E_s})^2}{N_0} \right\} dr_1. \]
\[ P[\text{error}] = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[ \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} e^{-x^2/2} \, dx \right]^{M-1} \exp \left[ -\frac{1}{2} \left( y - \sqrt{\frac{2 \log_2 M E_b}{N_0}} \right)^2 \right] \, dy. \]
Due to the symmetry of $M$-FSK constellation, all mappings from sequences of $\lambda$ bits to signal points yield the same bit error probability.

For equally likely signals, all the conditional error events are equiprobable and occur with probability
\[
\Pr[\text{symbol error}] / (M - 1) = \Pr[\text{symbol error}] / (2^\lambda - 1).
\]

There are \(\binom{\lambda}{k}\) ways in which $k$ bits out of $\lambda$ may be in error
\[\Rightarrow\] The average number of bit errors per $\lambda$-bit symbol is
\[
\sum_{k=1}^{\lambda} k \binom{\lambda}{k} \frac{\Pr[\text{symbol error}]}{2^\lambda - 1} = \lambda \frac{2^{\lambda-1}}{2^\lambda - 1} \Pr[\text{symbol error}].
\]

The probability of bit error is simply the above quantity divided by $\lambda$:
\[
\Pr[\text{bit error}] = \frac{2^{\lambda-1}}{2^\lambda - 1} \Pr[\text{symbol error}].
\]
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Union Bound on the Symbol Error Probability of \( M \)-FSK

\[
P[\text{error}] = P[(r_1 < r_2) \text{ or } (r_1 < r_3) \text{ or, } \cdots \text{, or } (r_1 < r_M) | s_1(t)].
\]

- Since the events are not mutually exclusive, the error probability is bounded by:

\[
P[\text{error}] < P[(r_1 < r_2) | s_1(t)] + P[(r_1 < r_3) | s_1(t)] + \cdots + P[(r_1 < r_M) | s_1(t)].
\]

- But \( P[(r_1 < r_j) | s_1(t)] = Q\left(\frac{\sqrt{E_s/N_0}}{}\right), j = 3, 4, \ldots, M. \)

Then

\[
P[\text{error}] < (M - 1)Q\left(\sqrt{E_s/N_0}\right) < MQ\left(\sqrt{E_s/N_0}\right) < Me^{-E_s/(2N_0)}.
\]

where the bound \( Q(x) < \exp\left\{-\frac{x^2}{2}\right\} \) has been used.
An Upper Bound on $Q(x)$

$$Q(x) = \int_x^\infty \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{\lambda^2}{2} \right\} d\lambda < \exp \left\{ -\frac{x^2}{2} \right\}$$
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Interpretations of $P[\text{error}] < Me^{-Es/(2N_0)}$

1. Let $M = 2^\lambda = e^{\lambda \ln 2}$ and $E_s = \lambda E_b$. Then

$$P[\text{error}] < e^{\lambda \ln 2} e^{-\lambda E_b/(2N_0)} = e^{-\lambda (E_b/N_0 - 2 \ln 2)/2}.$$ 

As $\lambda \to \infty$, or equivalently, as $M \to \infty$, the probability of error approaches zero exponentially, provided that

$$\frac{E_b}{N_0} > 2 \ln 2 = 1.39 = 1.42 \text{ dB}.$$ 

2. Since $E_s = \lambda E_b = V^2 T_s/2$, then

$$P[\text{error}] < e^{\lambda \ln 2} e^{-V^2 T_s/(4N_0)} = e^{-T_s[-r_b \ln 2 + V^2/(4N_0)]}$$

If $-r_b \ln 2 + V^2/(4N_0) > 0$, or $r_b < \frac{V^2}{4N_0 \ln 2}$, the probability or error tends to zero as $T_s$ or $M$ becomes larger and larger.
Comparison of $M$-ary Signaling Techniques

- A compact and meaningful comparison is based on the bit rate-to-bandwidth ratio, $r_b/W$ \textit{(bandwidth efficiency)} versus the SNR per bit, $E_b/N_0$ \textit{(power efficiency)} required to achieve a given $P[\text{error}]$.

- $M$-ASK with \textit{single-sideband} (SSB) transmission, $W = 1/(2T_s)$ and
  \[
  \left( \frac{r_b}{W} \right)_{\text{SSB-ASK}} = 2 \log_2 M \text{ (bits/s/Hz)}. 
  \]

- $M$-PSK ($M > 2$) must have \textit{double sidebands}, $W = 1/T_s$ and
  \[
  \left( \frac{r_b}{W} \right)_{\text{PSK}} = \log_2 M, \text{ (bits/s/Hz)}, 
  \]

- (Rectangular) QAM has twice the rate of ASK, but must have double sidebands $\Rightarrow$ QAM and SSB-ASK have the same bandwidth efficiency.

- For $M$-FSK with the minimum frequency separation of $1/(2T_s)$, $W = \frac{M}{2T_s} = \frac{M}{2(\lambda/r_b)} = \frac{M}{2 \log_2 M} r_b$, and
  \[
  \left( \frac{r_b}{W} \right)_{\text{FSK}} = \frac{2 \log_2 M}{M}. 
  \]
Chapter 8: *M*-ary Signaling Techniques

**USSB Transmission of BPSK Signal**

\[
\hat{m}(t) = m(t) * h(t) = m(t) * \left( -\frac{1}{\pi t} \right) = -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(t - \lambda)}{\lambda} d\lambda
\]
Example of USSB-BPSK Transmitted Signal

(a) BPSK signal

(b) USSB−BPSK signal
Chapter 8: $M$-ary Signaling Techniques

Power-Bandwidth Plane (At $P[\text{error}] = 10^{-5}$)

- $M=8$
- $M=16$
- $M=32$
- $M=64$

**Bandwidth-limited region:** $r_b/W > 1$

**Power-limited region:** $r_b/W < 1$

$M=8$

$M=16$

$M=32$

$M=64$

- PSK
- QAM and ASK (SSB)
- FSK

SNR per bit, $E_b/N_0$ (dB)

$r_b/W$ (bits/s/Hz)
Two Statements

Consider information transmission over an additive white Gaussian noise (AWGN) channel. The average transmitted signal power is $P_{av}$, the noise power spectral density is $N_0/2$ and the bandwidth is $W$. Two statements are:

1. For each transmission rate $r_b$, there is a corresponding limit on the probability of bit error one can achieve.

2. For some appropriate signalling rate $r_b$, there is no limit on the probability of bit error one can achieve, i.e., one can achieve error-free transmission.

Which statement sounds reasonable to you?
Shannon’s Channel Capacity

\[ C = W \log_2 \left( 1 + \frac{P_{av}}{W N_0} \right), \]

where \( W \) is bandwidth in Hz, \( P_{av} \) is the average power and \( N_0/2 \) is the two-sided power spectral density of the noise.

- Shannon proved that it is theoretically possible to transmit information at any rate \( r_b \), where \( r_b \leq C \), with an arbitrarily small error probability by using a sufficiently complicated modulation scheme. For \( r_b > C \), it is not possible to achieve an arbitrarily small error probability.

- Shannon’s work showed that the values of \( P_{av}, N_0 \) and \( W \) set a limit on transmission rate, not on error probability!

The normalized channel capacity \( C/W \) (bits/s/Hz) is:

\[ \frac{C}{W} = \log_2 \left( 1 + \frac{P_{av}}{W N_0} \right) = \log_2 \left( 1 + \frac{C}{W} \frac{E_b}{N_0} \right). \]
Shannon’s Capacity Curve
\[ \frac{E_b}{N_0} = \frac{2^{C/W} - 1}{C/W} \]

- **Channel capacity limit, \( C/W \)**
- **Shannon limit**
- **Bandwidth-limited region: \( r_b/W > 1 \)**
- **Power-limited region: \( r_b/W < 1 \)**

- PSK
- QAM and ASK (SSB)
- FSK
Chapter 8: M-ary Signaling Techniques

Spectrum Efficiency of DVB-S2 Standard

Example 1: 50Mbit/s in 36MHz at 4dB with QPSK

Example 2: 80 Mbit/s in 36MHz at 9.5dB with 8PSK

More information: www.dvb.org