A First Course in Digital Communications

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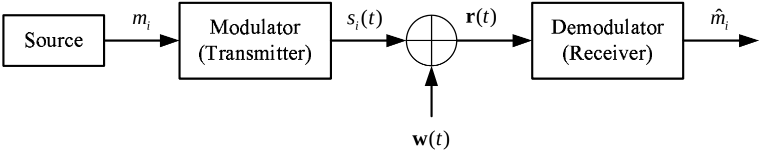
Introduction

- There are benefits to be gained when M-ary $(M = 4)$ signaling methods are used rather than straightforward binary signaling.
- In general, M-ary communication is used when one needs to design a communication system that is bandwidth efficient.
- Unlike QPSK and its variations, the gain in bandwidth is accomplished at the expense of error performance.
- To use M-ary modulation, the bit stream is blocked into groups of λ bits $\Rightarrow$ the number of bit patterns is $M = 2^{\lambda}$.
- The symbol transmission rate is $r_s = 1/T_s = 1/(\lambda T_b) = r_b/\lambda$ symbols/sec $\Rightarrow$ there is a bandwidth saving of $1/\lambda$ compared to binary modulation.
- Shall consider M-ary ASK, PSK, QAM (quadrature amplitude modulation) and FSK.

Optimum Receiver for M-ary Signaling



- $\mathbf{w}(t)$ is zero-mean white Gaussian noise with power spectral density of $\frac{N_0}{2}$ (watts/Hz).
- Receiver needs to make the decision on the transmitted signal based on the received signal $\mathbf{r}(t) = s_i(t) + \mathbf{w}(t)$.
- The determination of the optimum receiver (with minimum error) proceeds in a manner analogous to that for the binary case.

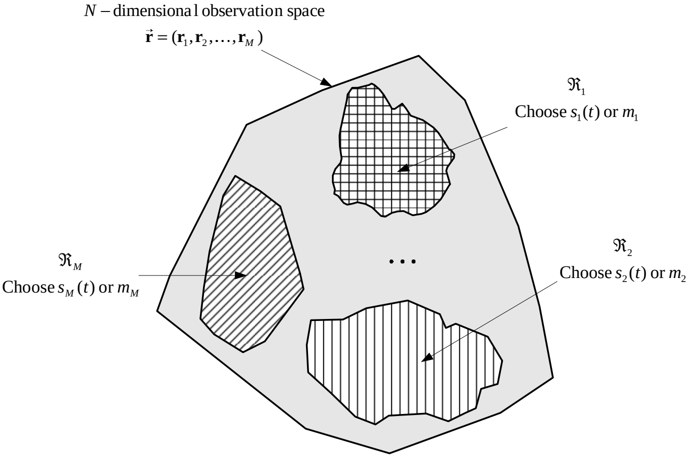
- Represent M signals by an orthonormal basis set, $\{\phi_n(t)\}_{n=1}^{N}$, $N \le M$:

$$\begin{aligned} s_i(t) &= s_{i1}\phi_1(t) + s_{i2}\phi_2(t) + \cdots + s_{iN}\phi_N(t), \\ s_{ik} &= \int_0^{T_s} s_i(t)\phi_k(t)\mathrm{d}t. \end{aligned}$$

- Expand the received signal $\mathbf{r}(t)$ into the series

$$\begin{aligned} \mathbf{r}(t) &= s_i(t) + \mathbf{w}(t) \\ &= \mathbf{r}_1\phi_1(t) + \mathbf{r}_2\phi_2(t) + \cdots + \mathbf{r}_N\phi_N(t) + \mathbf{r}_{N+1}\phi_{N+1}(t) + \cdots \end{aligned}$$

- For $k > N$, the coefficients $\mathbf{r}_k$ can be discarded.
- Need to partition the N-dimensional space formed by $\vec{\mathbf{r}} = (\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ into M regions so that the message error probability is minimized.



The optimum receiver is also the *minimum-distance receiver*:

Choose m_i if

$$\sum_{k=1}^{N}(r_k - s_{ik})^2 < \sum_{k=1}^{N}(r_k - s_{jk})^2;$$

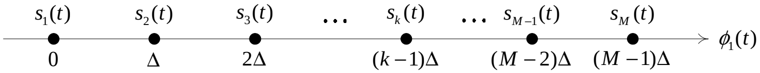
$$j = 1, 2, \ldots, M;\ j \neq i.$$

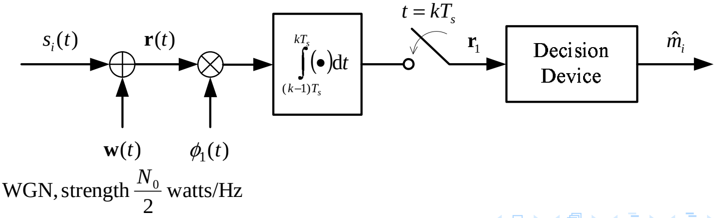
M-ary Coherent Amplitude-Shift Keying (M-ASK)

$$s_i(t) = V_i\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),\ 0 \le t \le T_s$$

$$= [(i-1)\Delta]\phi_1(t), \quad \phi_1(t) = \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),\ 0 \le t \le T_s,$$

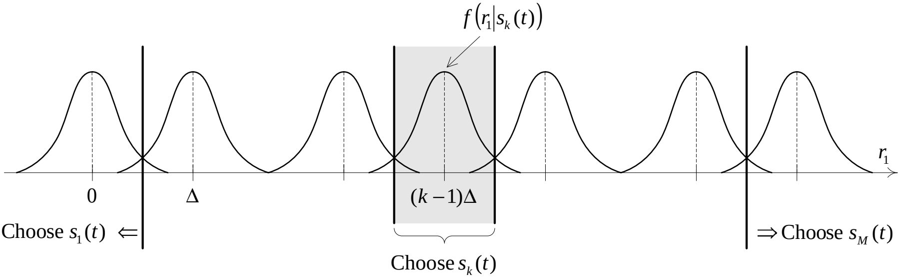
$$i = 1, 2, \ldots, M.$$



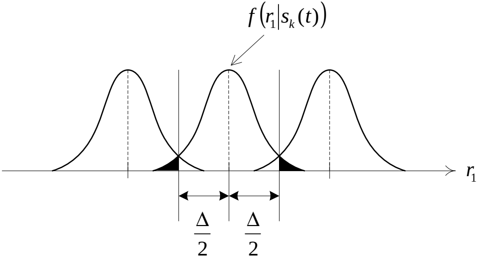


Minimum-Distance Decision Rule for M-ASK

$$\text{Choose} \begin{cases} s_k(t), & \text{if} \quad \left(k-\frac{3}{2}\right)\Delta < r_1 < \left(k-\frac{1}{2}\right)\Delta, \;\; k=2,3,\ldots,M-1 \\ s_1(t), & \text{if} \quad r_1 < \frac{\Delta}{2} \\ s_M(t), & \text{if} \quad r_1 > \left(M-\frac{3}{2}\right)\Delta \end{cases}.$$



Error Performance of M-ASK



$$P[\text{error}] = \sum_{i=1}^{M} P[s_i(t)] P[\text{error}|s_i(t)].$$

$$P[\text{error}|s_i(t)] = 2Q\left(\Delta/\sqrt{2N_0}\right), \quad i = 2, 3, \ldots, M-1.$$

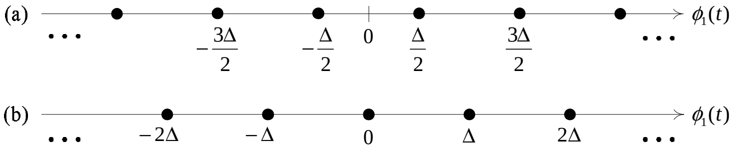
$$P[\text{error}|s_i(t)] = Q\left(\Delta/\sqrt{2N_0}\right), \quad i = 1, M.$$

$$P[\text{error}] = \frac{2(M-1)}{M} Q\left(\Delta/\sqrt{2N_0}\right).$$

Modified M-ASK Constellation

The maximum and average transmitted energies can be reduced, without any sacrifice in error probability, by changing the signal set to one which includes the negative version of each signal.

$$s_i(t) = \underbrace{(2i-1-M)\frac{\Delta}{2}}_{V_i}\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t),\ 0 \le t \le T_s,\ i = 1, 2, \ldots, M.$$



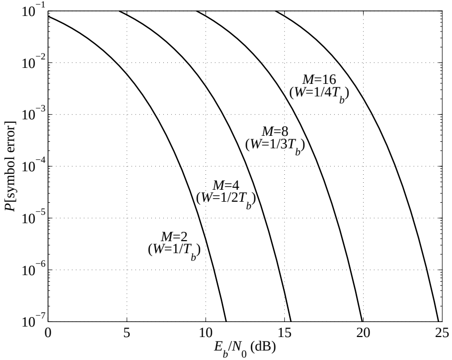
$$E_s = \frac{\sum_{i=1}^{M} E_i}{M} = \frac{\Delta^2}{4M}\sum_{i=1}^{M}(2i-1-M)^2 = \frac{(M^2-1)\Delta^2}{12}.$$

$$E_b = \frac{E_s}{\log_2 M} = \frac{(M^2-1)\Delta^2}{12\log_2 M} \Rightarrow \Delta = \sqrt{\frac{(12\log_2 M)E_b}{M^2-1}}$$

Probability of Symbol Error for M-ASK

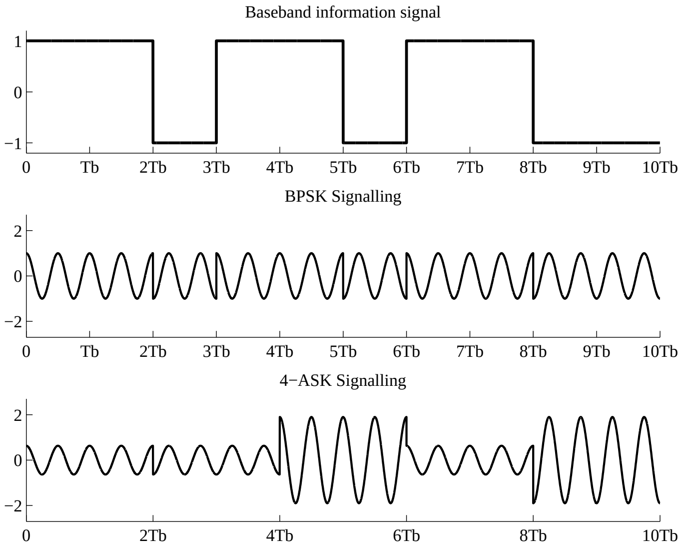
$$P[\text{error}] = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6E_s}{(M^2-1)N_0}}\right) = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\log_2 M}{M^2-1}\frac{E_b}{N_0}}\right).$$

$$P[\text{bit error}] = \frac{1}{\lambda} P[\text{symbol error}] = \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{6\log_2 M}{M^2-1}\frac{E_b}{N_0}}\right) \text{ (with Gray mapping)}$$



W is obtained by using the $WT_s = 1$ rule-of-thumb. Here $1/T_b$ is the bit rate (bits/s).

Example of 2-ASK (BPSK) and 4-ASK Signals



M-ary Phase-Shift Keying (M-PSK)

$$s_i(t) = V\cos\left[2\pi f_c t - \frac{(i-1)2\pi}{M}\right], \quad 0 \le t \le T_s,$$

$i = 1, 2, \ldots, M;\ f_c = k/T_s,\ k$ integer; $E_s = V^2 T_s/2$ joules

$$s_i(t) = V\cos\left[\frac{(i-1)2\pi}{M}\right]\cos(2\pi f_c t) + V\sin\left[\frac{(i-1)2\pi}{M}\right]\sin(2\pi f_c t).$$

$$\phi_1(t) = \frac{V\cos(2\pi f_c t)}{\sqrt{E_s}},\ \phi_2(t) = \frac{V\sin(2\pi f_c t)}{\sqrt{E_s}}.$$

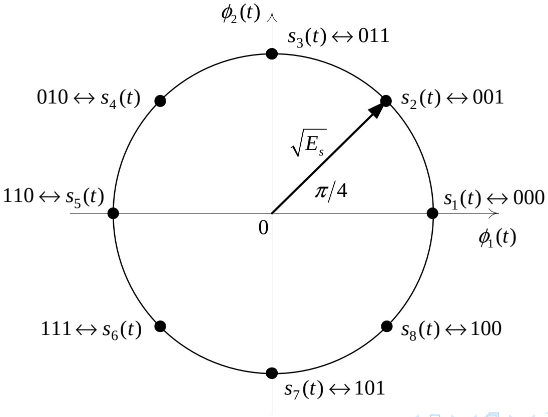
$$s_{i1} = \sqrt{E_s}\cos\left[\frac{(i-1)2\pi}{M}\right],\ s_{i2} = \sqrt{E_s}\sin\left[\frac{(i-1)2\pi}{M}\right].$$

The signals lie on a circle of radius $\sqrt{E_s}$, and are spaced every $2\pi/M$ radians around the circle.

Signal Space Plot of 8-PSK

$$s_i(t) = V\cos\left[2\pi f_c t - \frac{(i-1)2\pi}{M}\right], \quad 0 \le t \le T_s,$$

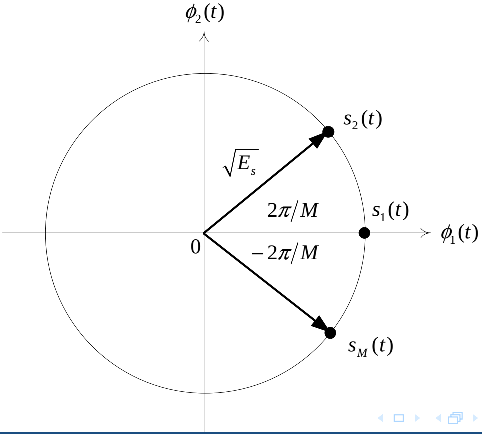
$i = 1, 2, \ldots, M$; $f_c = k/T_s$, k integer; $E_s = V^2 T_s/2$ joules



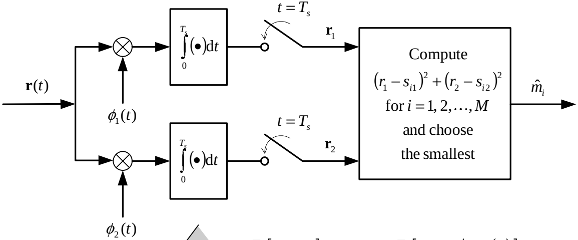
Signal Space Plot of General M-PSK

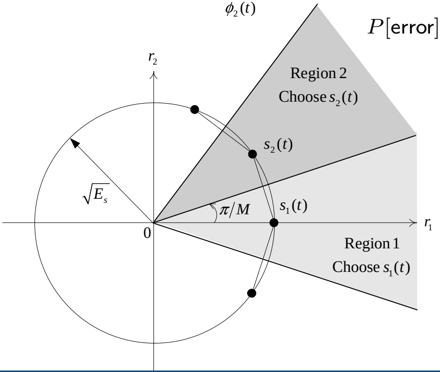
$$s_i(t) = V\cos\left[2\pi f_c t - \frac{(i-1)2\pi}{M}\right], \quad 0 \le t \le T_s,$$

$i = 1, 2, \ldots, M$; $f_c = k/T_s$, k integer; $E_s = V^2 T_s/2$ joules



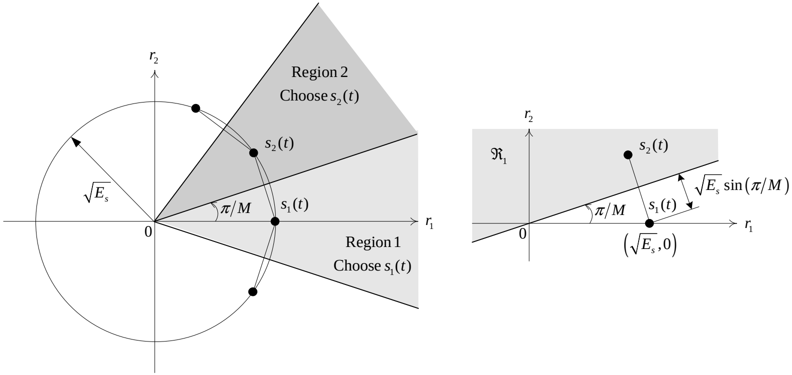
Optimum Receiver for M-PSK





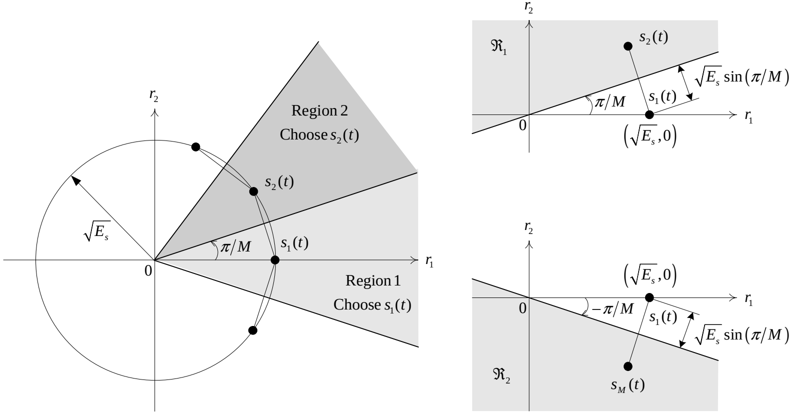
$$
\begin{aligned}
P[\text{error}] &= P[\text{error}|s_1(t)] \\
&= 1 - \iint_{r_1, r_2 \in \text{Region 1}} f(r_1, r_2|s_1(t)) \mathrm{d}r_1 \mathrm{d}r_2.
\end{aligned}
$$

Lower Bound of P[error] of M-PSK



$$
\begin{aligned}
P[\text{error}|s_1(t)] &> P[r_1, r_2 \text{ fall in } \Re_1 | s_1(t)], \text{ or} \\
P[\text{error}|s_1(t)] &> Q\left\{\sin\left(\frac{\pi}{M}\right)\sqrt{2E_s/N_0}\right\}.
\end{aligned}
$$

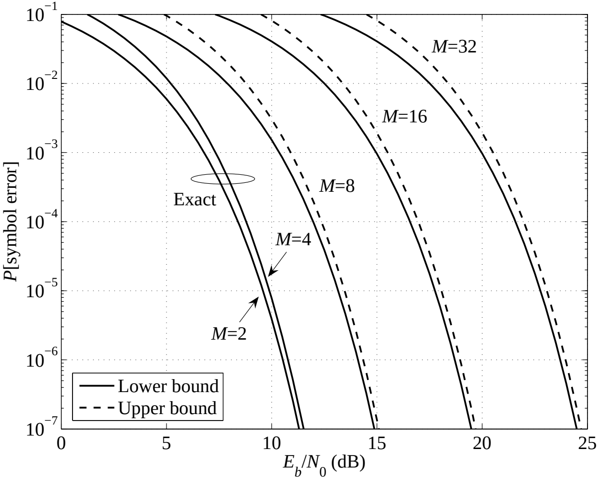
Upper Bound of $P[\text{error}]$ of M-PSK



$$P[\text{error}] < P[r_1, r_2 \text{ fall in } \Re_1 | s_1(t)] + P[r_1, r_2 \text{ fall in } \Re_2 | s_1(t)], \text{ or}$$

$$P[\text{error}] < 2Q\left(\sin\left(\frac{\pi}{M}\right)\sqrt{2E_s/N_0}\right),$$

Symbol Error Probability of M-PSK



With a Gray mapping, the *bit* error probability is approximated as:

$$P[\text{bit error}]_{M\text{-PSK}} \simeq \frac{1}{\log_2 M} Q\left(\sqrt{\lambda \sin^2\left(\frac{\pi}{M}\right)\frac{2E_b}{N_0}}\right).$$

Comparison of BPSK and M-PSK

$$P[\text{error}]_{M\text{-PSK}} \simeq Q\left(\sqrt{\lambda \sin^2\left(\frac{\pi}{M}\right)\frac{2E_b}{N_0}}\right), \quad \text{where } E_s = \lambda E_b.$$

$$P[\text{error}]_{\text{BPSK}} = Q(\sqrt{2E_b/N_0}).$$

λ	M	M-ary BW/Binary BW	$\lambda \sin^2(\pi/M)$	M-ary Energy/Binary Energy
3	8	1/3	0.44	3.6 dB
4	16	1/4	0.15	8.2 dB
5	32	1/5	0.05	13.0 dB
6	64	1/6	0.0144	17.0 dB

M-ary Quadrature Amplitude Modulation (M-QAM)

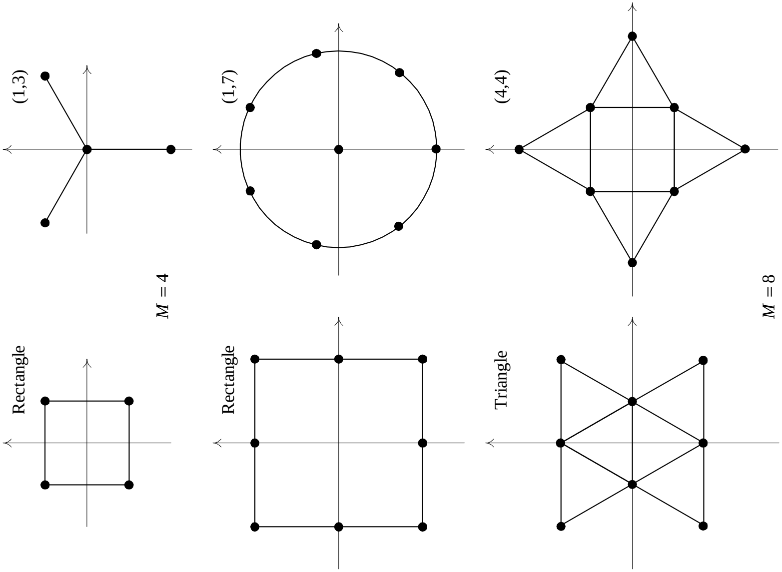
- M-QAM constellations are two-dimensional and they involve inphase (I) and quadrature (Q) carriers:

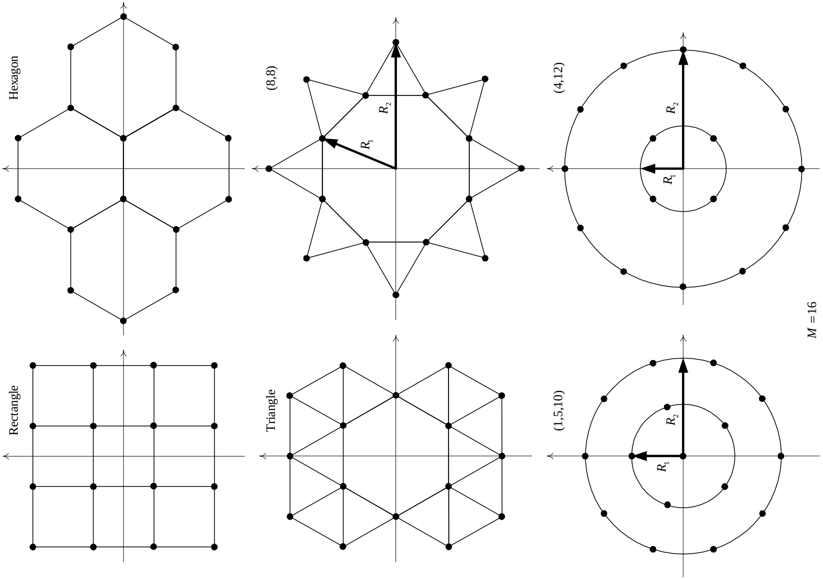
$$\begin{aligned} \phi_I(t) &= \sqrt{\frac{2}{T_s}}\cos(2\pi f_c t), \quad 0 \le t \le T_s, \\ \phi_Q(t) &= \sqrt{\frac{2}{T_s}}\sin(2\pi f_c t), \quad 0 \le t \le T_s, \end{aligned}$$

- The ith transmitted M-QAM signal is:

$$\begin{aligned} s_i(t) &= V_{I,i}\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t) + V_{Q,i}\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t), \quad \begin{array}{l} 0 \le t \le T_s \\ i = 1, 2, \ldots, M \end{array} \\ &= \sqrt{E_i}\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t - \theta_i) \end{aligned}$$

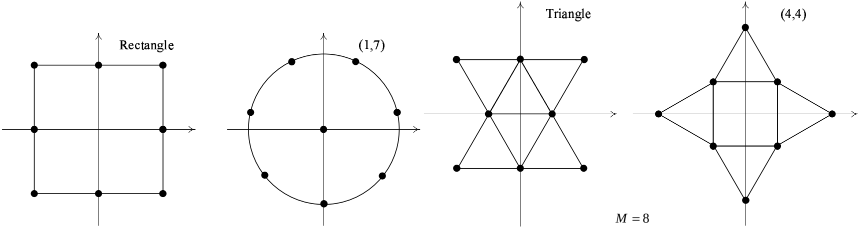
$V_{I,i}$ and $V_{Q,i}$ are the information-bearing discrete amplitudes of the two quadrature carriers, $E_i = V_{I,i}^2 + V_{Q,i}^2$ and $\theta_i = \tan^{-1}(V_{Q,i}/V_{I,i})$.


(1,3)
(1,7)
(4,4)
M = 4
M = 8
Rectangle
Rectangle
Triangle


Hexagon
(8,8)
R2
R1
(4,12)
R2
R1
M = 16
Rectangle
Triangle
(1,5,10)
R2
R1

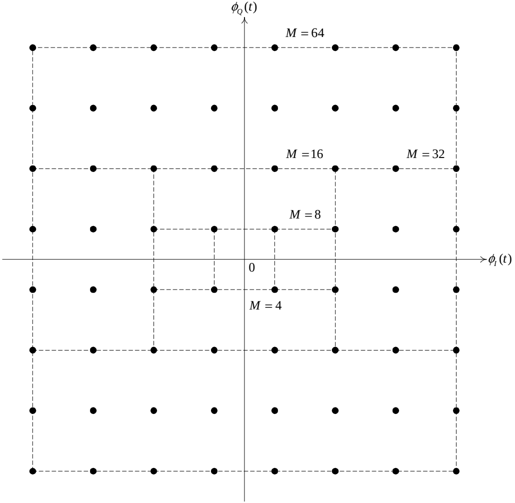
A Simple Comparison of M-QAM Constellations

With the same *minimum* distance of all the constellations, a more efficient signal constellation is the one that has smaller average transmitted energy.



E_s for the rectangular, triangular, (1,7) and (4,4) constellations are found to be $1.50\Delta^2$, $1.125\Delta^2$, $1.162\Delta^2$ and $1.183\Delta^2$, respectively.

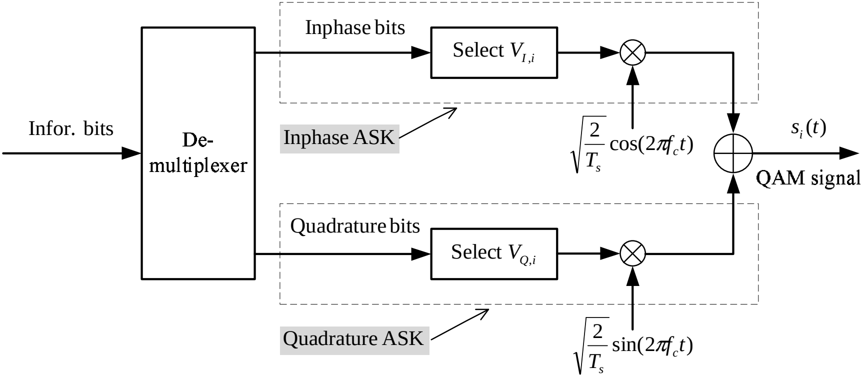
Rectangular M-QAM



The signal *components* take value from the set of discrete values $\{(2i - 1 - M)\Delta/2\},\ i = 1, 2, \ldots, \frac{M}{2}$.

Modulation of Rectangular M-QAM

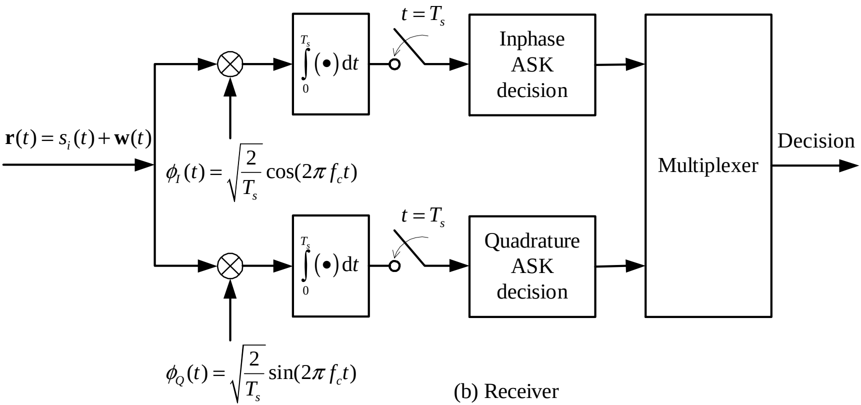
- Each group of $\lambda = \log_2 M$ bits can be divided into λ_I inphase bits and λ_Q quadrature bits, where $\lambda_I + \lambda_Q = \lambda$.
- Inphase bits and quadrature bits modulate the inphase and quadrature carriers *independently*.



(a) Transmitter

Demodulation of Rectangular M-QAM

Due to the orthogonality of the inphase and quadrature signals, inphase and quadrature bits can be *independently* detected at the receiver.



(b) Receiver

The most practical rectangular QAM constellation is one which $\lambda_I = \lambda_Q = \lambda/2$, i.e., M is a perfect square and the rectangle is a square.

Symbol Error Probability of M-QAM

- For square constellations:

$$P[\text{error}] = 1 - P[\text{correct}] = 1 - \left(1 - P_{\sqrt{M}}[\text{error}]\right)^2,$$

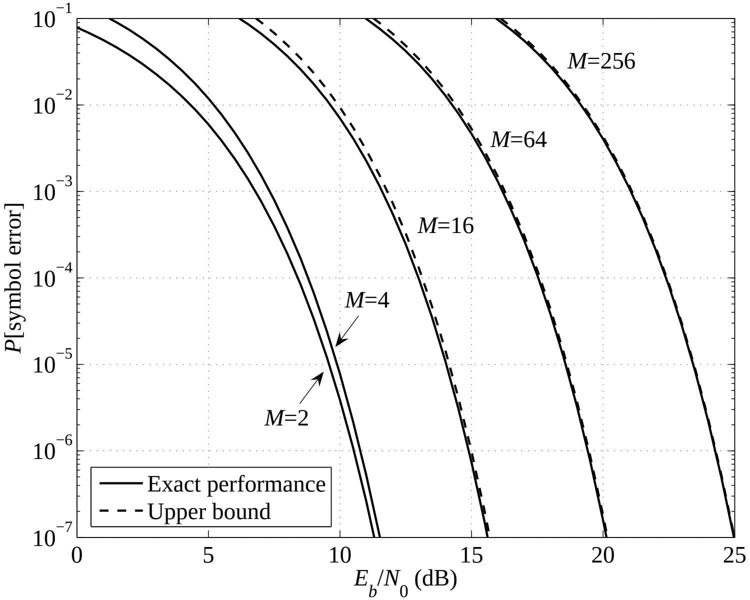
$$P_{\sqrt{M}}[\text{error}] = 2\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right),$$

where E_s/N_0 is the average SNR per symbol.

- For general rectangular constellations:

$$\begin{aligned} P[\text{error}] &\leq 1 - \left[1 - 2Q\left(\sqrt{\frac{3E_s}{(M-1)N_0}}\right)\right]^2 \\ &\leq 4Q\left(\sqrt{\frac{3\lambda E_b}{(M-1)N_0}}\right) \end{aligned}$$

where E_b/N_0 is the average SNR per bit.


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
P[symbol error]
0
5
10
15
20
25
E_b/N_0 (dB)
M=256
M=64
M=16
M=4
M=2
Exact performance
Upper bound

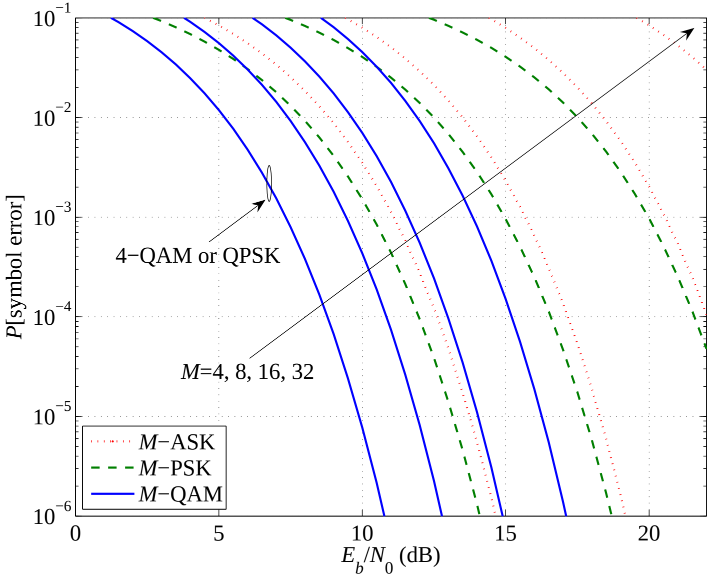
Performance Comparison of M-PSK and M-QAM

- For M-PSK, approximate $P[\text{error}] \approx Q\left(\sqrt{\frac{2E_s}{N_0}} \sin \frac{\pi}{M}\right)$.
- For M-QAM, use the upper bound $4Q\left(\sqrt{\frac{3\lambda E_b}{(M-1)N_0}}\right)$.
- Comparing the arguments of $Q(\cdot)$ for the two modulations gives:

$$\kappa_M = \frac{3/(M-1)}{2\sin^2(\pi/M)}.$$

M	$10\log_{10}\kappa_M$
8	1.65 dB
16	4.20 dB
32	7.02 dB
64	9.95 dB
256	15.92 dB
1024	21.93 dB

Performance Comparison of M-ASK, M-PSK, M-QAM

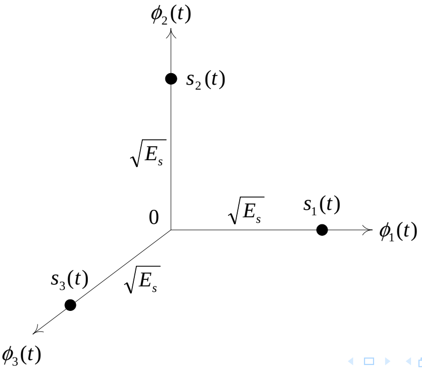


M-ary Coherent Frequency-Shift Keying (M-FSK)

$$s_i(t) = \begin{cases} V\cos(2\pi f_i t), & 0 \le t \le T_s \\ 0, & \text{elsewhere} \end{cases}, \quad i = 1, 2, \ldots, M,$$

where f_i are chosen to have orthogonal signals over $[0, T_s]$.

$$f_i = \begin{cases} (k \pm i)\left(\frac{1}{2T_s}\right), & \text{(coherently orthogonal)} \\ (k \pm i)\left(\frac{1}{T_s}\right), & \text{(noncoherently orthogonal)} \end{cases}, \quad i = 0, 1, 2, \ldots$$



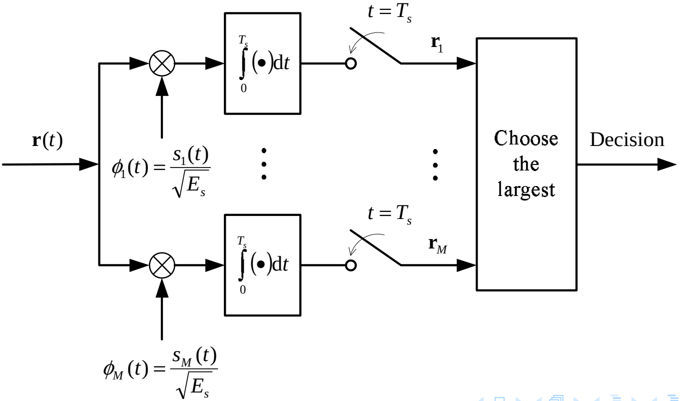
Minimum-Distance Receiver of M-FSK

Choose m_i if

$$\sum_{k=1}^{M}(r_k - s_{ik})^2 < \sum_{k=1}^{M}(r_k - s_{jk})^2 \quad \Rightarrow$$
$$j = 1, 2, \ldots, M; \; j \neq i,$$

Choose m_i if

$$r_i > r_j, \quad j = 1, 2, \ldots, M; \; j \neq i.$$



Symbol Error Probability of M-FSK

$$P[\text{error}] = P[\text{error}|s_1(t)] = 1 - P[\text{correct}|s_1(t)].$$

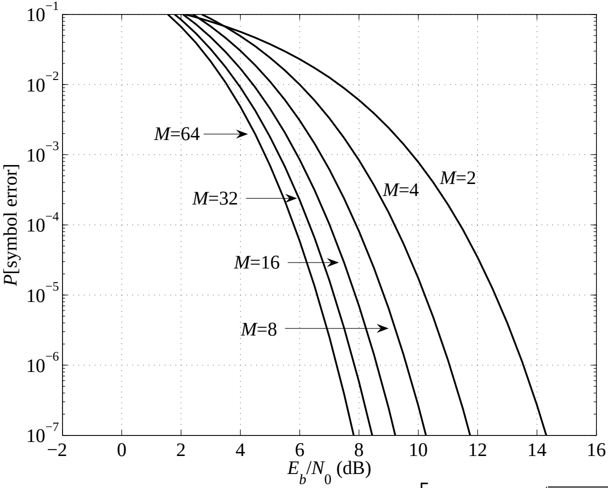
$$P[\text{correct}|s_1(t)] = P[(\mathbf{r}_2 < \mathbf{r}_1) \text{ and } \cdots \text{ and } (\mathbf{r}_M < \mathbf{r}_1)|s_1(t) \text{ sent}].$$

$$= \int_{r_1=-\infty}^{\infty} P[(\mathbf{r}_2 < r_1) \text{ and} \cdots \text{and } (\mathbf{r}_M < r_1)|\{\mathbf{r}_1 = r_1, s_1(t)\}] f(r_1|s_1(t))\mathrm{d}r.$$

$$P[(\mathbf{r}_2 < r_1) \text{ and} \cdots \text{and} (\mathbf{r}_M < r_1)|\{\mathbf{r}_1 = r_1, s_1(t)\}] = \prod_{j=2}^{M} P[(\mathbf{r}_j < r_1)|\{\mathbf{r}_1 = r_1, s_1(t)\}]$$

$$P[\mathbf{r}_j < r_1|\{\mathbf{r}_1 = r_1, s_1(t)\}] = \int_{-\infty}^{r_1} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{\lambda^2}{N_0}\right\} \mathrm{d}\lambda.$$

$$P[\text{correct}] = \int_{r_1=-\infty}^{\infty} \left[\int_{\lambda=-\infty}^{r_1} \frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{\lambda^2}{N_0}\right\} \mathrm{d}\lambda\right]^{M-1} \times$$
$$\frac{1}{\sqrt{\pi N_0}} \exp\left\{-\frac{(r_1 - \sqrt{E_s})^2}{N_0}\right\} \mathrm{d}r_1.$$



$$P[\text{error}] = 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{y} \mathrm{e}^{-x^2/2} \mathrm{d}x \right]^{M-1} \exp\left[-\frac{1}{2} \left(y - \sqrt{\frac{2 \log_2 M E_b}{N_0}} \right)^2 \right] \mathrm{d}y.$$

Bit Error Probability of M-FSK

- Due to the symmetry of M-FSK constellation, all mappings from sequences of λ bits to signal points yield the same bit error probability.
- For equally likely signals, all the conditional error events are equiprobable and occur with probability $\Pr[\text{symbol error}]/(M-1) = \Pr[\text{symbol error}]/(2^\lambda - 1)$.
- There are $\binom{\lambda}{k}$ ways in which k bits out of λ may be in error $\Rightarrow$ The average number of bit errors per λ-bit symbol is

$$\sum_{k=1}^{\lambda} k \binom{\lambda}{k} \frac{\Pr[\text{symbol error}]}{2^\lambda - 1} = \lambda \frac{2^{\lambda-1}}{2^\lambda - 1} \Pr[\text{symbol error}].$$

- The probability of bit error is simply the above quantity divided by λ:

$$\Pr[\text{bit error}] = \frac{2^{\lambda-1}}{2^\lambda - 1} \Pr[\text{symbol error}].$$

Union Bound on the Symbol Error Probability of M-FSK

$$P[\text{error}] = P[(\mathbf{r}_1 < \mathbf{r}_2) \text{ or } (\mathbf{r}_1 < \mathbf{r}_3) \text{ or}, \cdots, \text{or } (\mathbf{r}_1 < \mathbf{r}_M)|s_1(t)].$$

- Since the events are not mutually exclusive, the error probability is bounded by:

$$P[\text{error}] < P[(\mathbf{r}_1 < \mathbf{r}_2)|s_1(t)] + P[(\mathbf{r}_1 < \mathbf{r}_3)|s_1(t)] + \cdots + P[(\mathbf{r}_1 < \mathbf{r}_M)|s_1(t)].$$

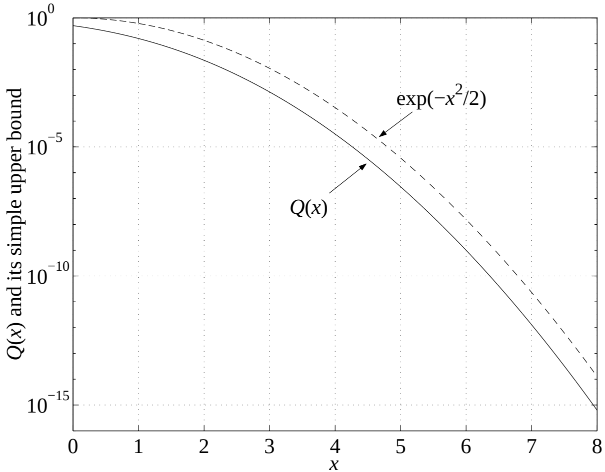
- But $P[(\mathbf{r}_1 < \mathbf{r}_j)|s_1(t)] = Q\left(\sqrt{E_s/N_0}\right)$, $j = 3, 4, \ldots, M$. Then

$$P[\text{error}] < (M-1)Q\left(\sqrt{E_s/N_0}\right) < MQ\left(\sqrt{E_s/N_0}\right) < M\mathrm{e}^{-E_s/(2N_0)}.$$

where the bound $Q(x) < \exp\left\{-\frac{x^2}{2}\right\}$ has been used.

An Upper Bound on $Q(x)$

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{\lambda^2}{2}\right\} \mathrm{d}\lambda < \exp\left\{-\frac{x^2}{2}\right\}$$



Interpretations of $P[\text{error}] < M\mathrm{e}^{-E_s/(2N_0)}$

1. Let $M = 2^{\lambda} = \mathrm{e}^{\lambda \ln 2}$ and $E_s = \lambda E_b$. Then

$$P[\text{error}] < \mathrm{e}^{\lambda \ln 2} \mathrm{e}^{-\lambda E_b/(2N_0)} = \mathrm{e}^{-\lambda(E_b/N_0 - 2\ln 2)/2}.$$

As $\lambda \to \infty$, or equivalently, as $M \to \infty$, the probability of error *approaches zero* exponentially, provided that

$$\frac{E_b}{N_0} > 2\ln 2 = 1.39 = 1.42 \text{ dB}.$$

2. Since $E_s = \lambda E_b = V^2 T_s/2$, then

$$P[\text{error}] < \mathrm{e}^{\lambda \ln 2} \mathrm{e}^{-V^2 T_s/(4N_0)} = \mathrm{e}^{-T_s[-r_b \ln 2 + V^2/(4N_0)]}$$

If $-r_b \ln 2 + V^2/(4N_0) > 0$, or $r_b < \frac{V^2}{4N_0 \ln 2}$ the probability or error tends to zero as T_s or M becomes larger and larger.

Comparison of M-ary Signaling Techniques

- A compact and meaningful comparison is based on the bit rate-to bandwidth ratio, r_b/W (*bandwidth efficiency*) versus the SNR per bit, E_b/N_0 (*power efficiency*) required to achieve a given $P[\text{error}]$.
- M-ASK with *single-sideband* (SSB) transmission, $W = 1/(2T_s)$ and

$$\left(\frac{r_b}{W}\right)_{\text{SSB-ASK}} = 2\log_2 M \quad \text{(bits/s/Hz)}.$$

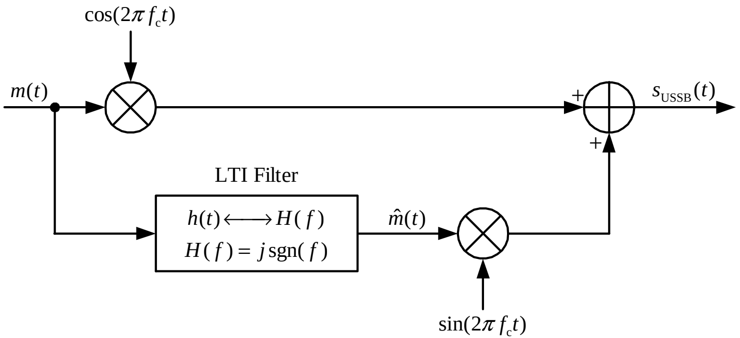
- M-PSK $(M > 2)$ must have *double sidebands*, $W = 1/T_s$ and

$$\left(\frac{r_b}{W}\right)_{\text{PSK}} = \log_2 M, \quad \text{(bits/s/Hz)},$$

- (Rectangular) QAM has twice the rate of ASK, but must have double sidebands $\Rightarrow$ QAM and SSB-ASK have the same bandwidth efficiency.
- For M-FSK with the minimum frequency separation of $1/(2T_s)$, $W = \frac{M}{2T_s} = \frac{M}{2(\lambda/r_b)} = \frac{M}{2\log_2 M} r_b$, and

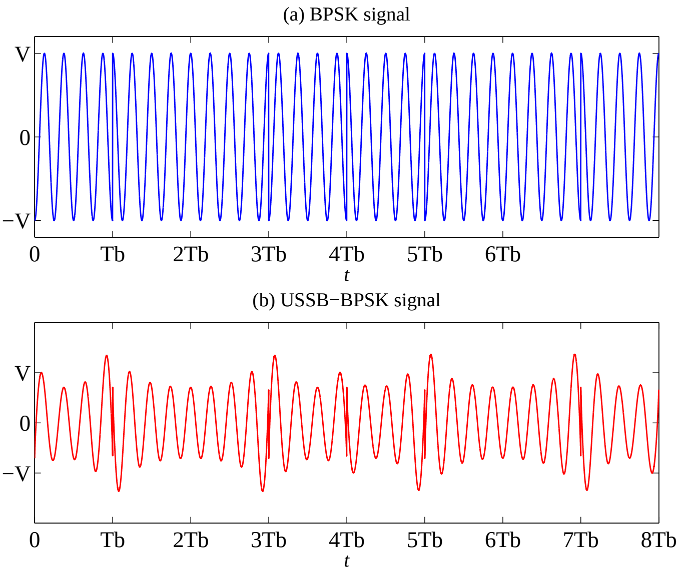
$$\left(\frac{r_b}{W}\right)_{\text{FSK}} = \frac{2\log_2 M}{M}.$$

USSB Transmission of BPSK Signal

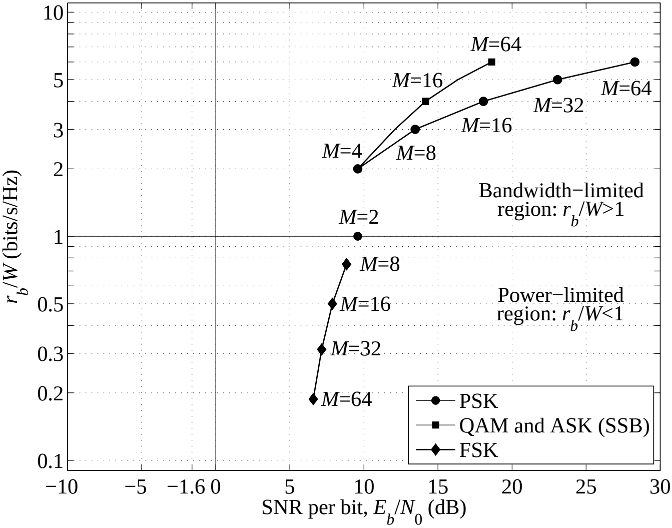


$$\hat{m}(t) = m(t) * h(t) = m(t) * \left(-\frac{1}{\pi t}\right) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{m(t-\lambda)}{\lambda}\mathrm{d}\lambda$$

Example of USSB-BPSK Transmitted Signal



Power-Bandwidth Plane (At $P[\text{error}] = 10^{-5}$)



Two Statements

Consider information transmission over an additive white Gaussian noise (AWGN) channel. The average transmitted signal power is P_{av}, the noise power spectral density is $N_0/2$ and the bandwidth is W. Two statements are:

1. For each transmission rate r_b, there is a corresponding limit on the *probability of bit error* one can achieve.
2. For some appropriate signalling rate r_b, there is no limit on the *probability of bit error* one can achieve, i.e., one can achieve error-free transmission.

Which statement sounds reasonable to you?

Shannon's Channel Capacity

$$C = W \log_2 \left(1 + \frac{P_{\text{av}}}{W N_0}\right),$$

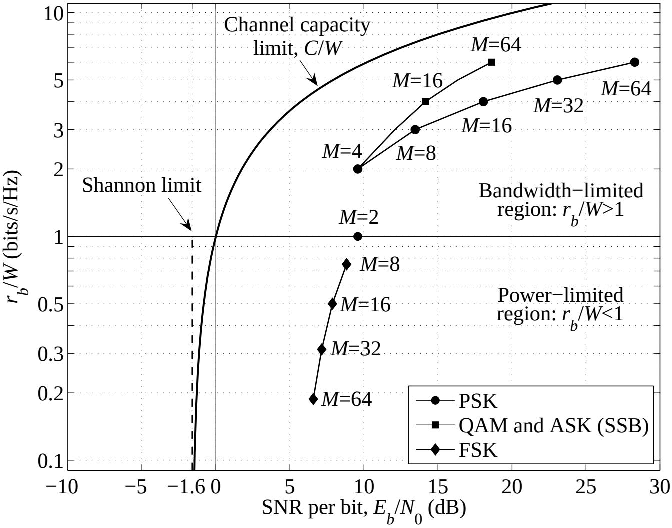
where W is bandwidth in Hz, P_{av} is the average power and $N_0/2$ is the two-sided power spectral density of the noise.

- Shannon proved that it is theoretically possible to transmit information at any rate r_b, where $r_b \leq C$, with an *arbitrarily small* error probability by using a sufficiently complicated modulation scheme. For $r_b > C$, it is not possible to achieve an arbitrarily small error probability.
- Shannon's work showed that the values of P_{av}, N_0 and W *set a limit on transmission rate, not on error probability*!

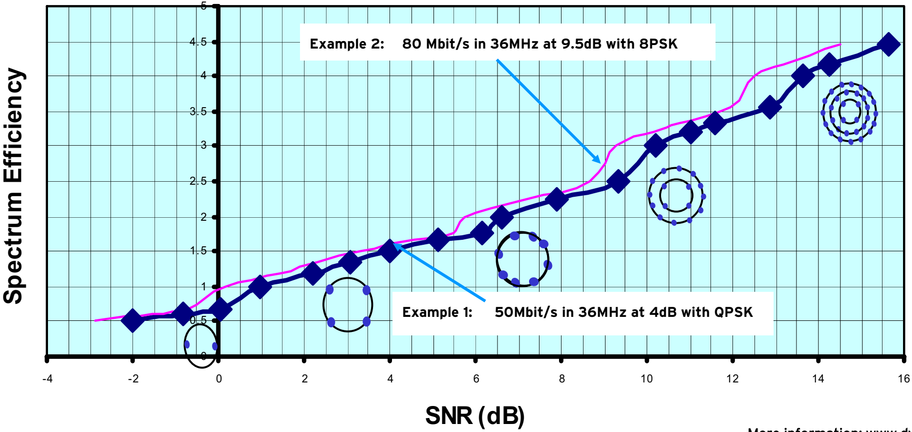
The normalized channel capacity C/W (bits/s/Hz) is:

$$\frac{C}{W} = \log_2 \left(1 + \frac{P_{\text{av}}}{W N_0}\right) = \log_2 \left(1 + \frac{C}{W} \frac{E_b}{N_0}\right).$$

Shannon's Capacity Curve $\frac{E_b}{N_0} = \frac{2^{C/W}-1}{C/W}$



Spectrum Efficiency of DVB-S2 Standard



More information: www.dvb.org