Wireless Communications

Chapter 5

Mobile Radio Propagation –

Small-Scale Fading and Multipath

- Small-scale fading, simply fading, is used to describe the rapid fluctuations of the amplitudes, phases, or multipath delays of a radio signal over a short period of time or travel distance.
- Fading is caused by interference between multipath waves. The resultant signal varies widely in amplitude and phase.

5.1 Small-Scale Multipath Propagation

- Multipath in the radio channel creates small-scale fading effects.
- The three most important effects are :
  1. Rapid changes in signal strength over a small travel distance or time interval.
  2. Random frequency modulation due to varying Doppler shifts on different multipath signals.
  3. Time dispersion (echoes) caused by multipath propagation delays.

- The signal received by the mobile at any point in space may consist of a large number of plane waves having randomly distributed amplitude, phase, and angles of arrival.
- The received signal may fade due to movement of surrounding objects in the radio channel.
- If the objects in the radio channel are static, the spatial variations of the resulting signal can be seen as temporal variation by the receiver.
- Avoid a deep fade in a long time interval.
- Multipath waves experience shifts in frequency due to Doppler effects.

5.1.1 Factors Influencing Small-Scale Fading

- Multipath propagation – Induce small-scale fading, signal distortion, or both. Signal distortion (smearing) is due to intersymbol interference.
- Speed of the mobile – Doppler shift.
- Speed of surrounding objects – The speed of the mobile or surrounding objects dominates the small-scale fading. The coherence time defines the "staticness" of the channel, and is directly impacted by the Doppler shift.

- The transmission bandwidth of the signal –

  (1) Coherence bandwidth of the channel is a measure of the maximum frequency difference for which signals are still strongly correlated in amplitude (denote as B_c).

  (2) If $B_{transmitted} > B_c$, the signal will be distorted, but the received signal strength will not fade much (significantly) over a local area. If the transmitted signal have narrow bandwidth as compared to channel, the amplitude of the signal will change rapidly, but the signal will not be distorted in time.

  (3) Signal strength and the likelihood of signal smearing are very much related to the multipath channel, as well as the bandwidth of the transmitted signal.

5.1.2 Doppler shift

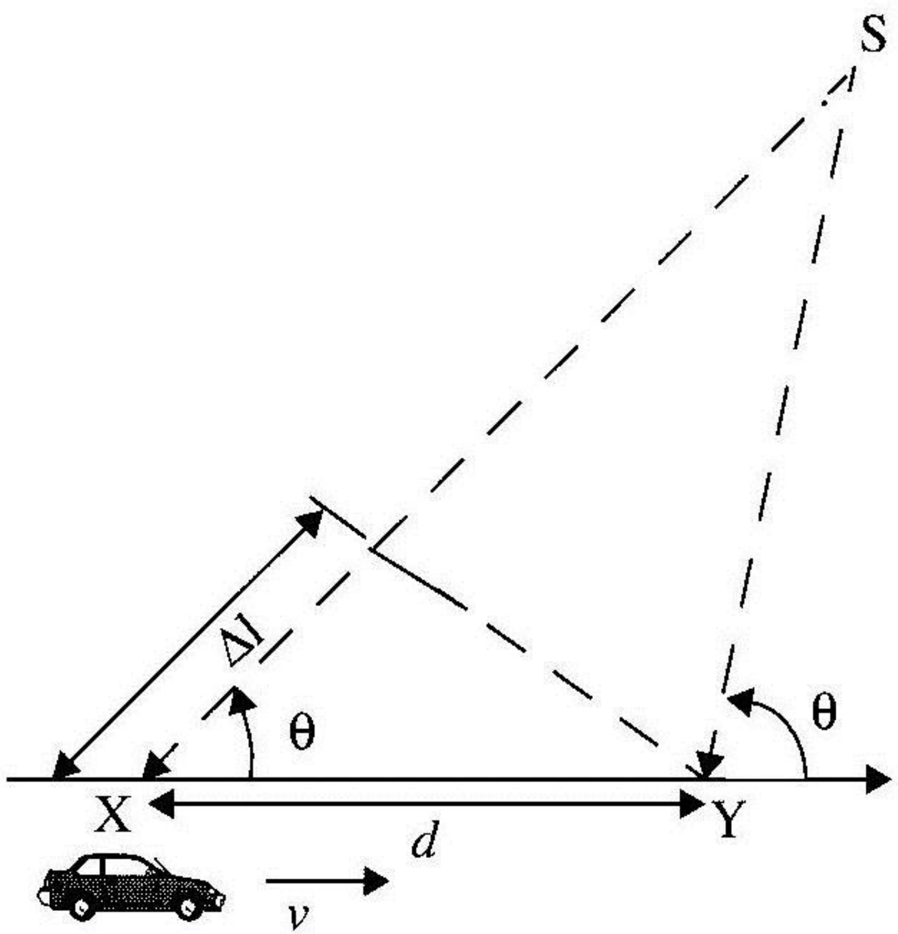


Figure 5.1 Illustration of Doppler effect.

- The phase change $\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$.
- Doppler shift $f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$.

  relating f_d to the mobile velocity v and the spatial angle θ.
- Multipath components from a CW signal that arrives from different directions contribute to Doppler spreading of the received signal, thus increasing the signal bandwidth.

Example 5.1

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving (a) directly toward the transmitter, (b) directly away from the transmitter, and (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

Solution:

Carrier frequency f_c = 1850 MHz,

wavelength $\lambda = c / f_c = (3\times10^8) / (1850 \times10^6)$

v = 60mph = 26.82 m/s

(a) $f = f_c + f_d = 1850 \times10^6 + \frac{26.82}{0.162}$ = 1850.00016 MHz

(b) $f = f_c - f_d = 1850 \times10^6 - \frac{26.82}{0.162}$ = 1849.999834 MHz

(c) In this case, θ = 90°, there is no Doppler shift.

5.2 Impulse Response Model of a Multipath Channel

- A mobile radio channel may be modeled as a linear filter with time varying impulse response, where the time variation is due to receiver motion in space.
- Consider the case where time variation is due to strictly to receiver motion in space.

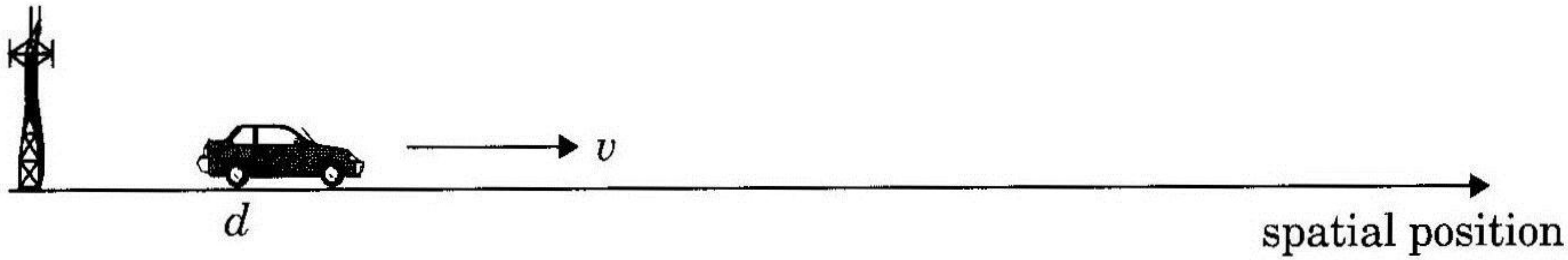


Figure 5.2 The mobile radio channel as a function of time and space.

$x(t)$: the transmitted signal.

$y(d,t)$: the received signal at position d.

$h(d,t)$: the channel impulse response at position d.

$$y(d,t) = x(t) \otimes h(d,t)$$
$$= \int_{-\infty}^{\infty} x(\tau) h(d, t-\tau) d\tau$$

- For a causal system, $h(d,t) = 0$ for $t < 0$

$$y(d,t) = \int_{-\infty}^{t} x(\tau)h(d,t-\tau)d\tau$$

$$d = vt$$

$$y(vt,t) = \int_{-\infty}^{t} x(\tau)h(vt,t-\tau)d\tau$$

$$= x(t) \otimes h(d,t) = x(t) \otimes h(vt,t)$$

$h(vt,t)$ = a linear time varying channel.

- $H(t,\tau)$ completely characterizes the channel, where t represents the time variations due to motion and τ represents the channel multipath delay for a fixed value of t .

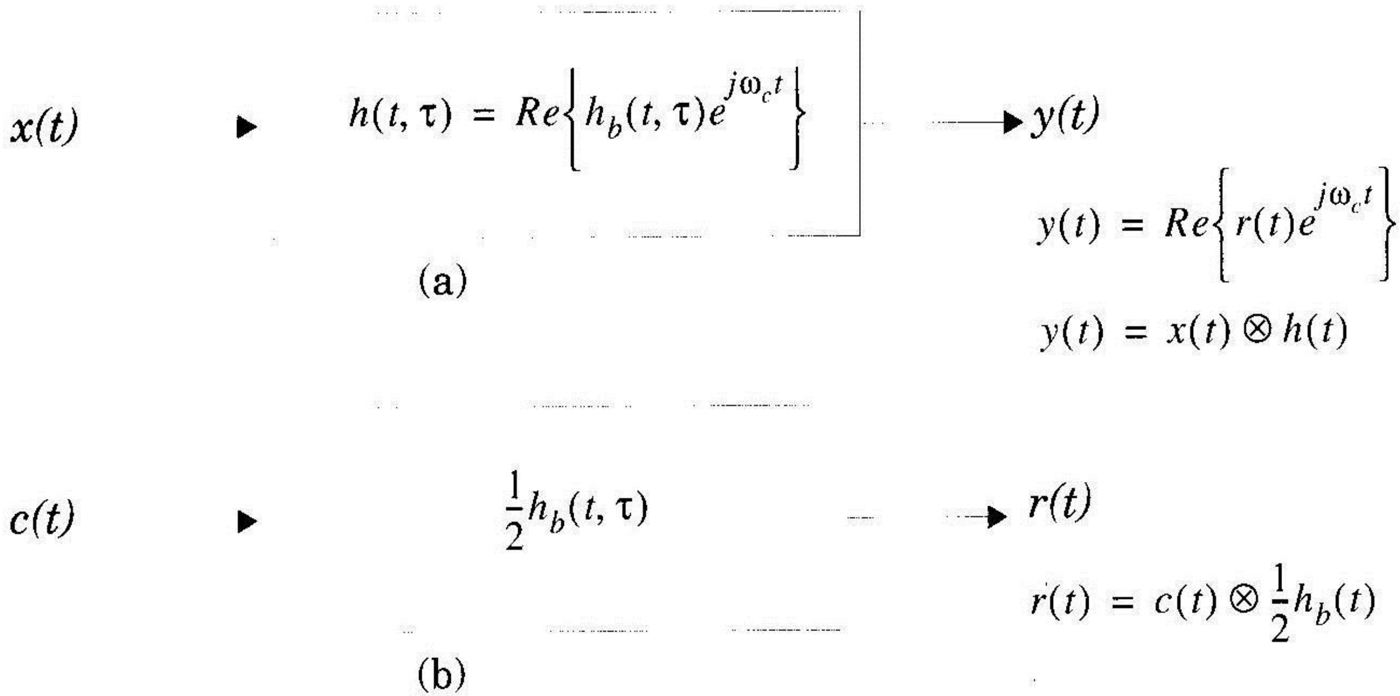


Figure 5.3 (a) Bandpass channel impulse response model; (b) baseband equivalent channel impulse response model.

- Passband $\longleftrightarrow$ Baseband ($r(t) = c(t) \otimes (1/2) h_b(t,\tau)$) .
- It is useful to discretize the multipath delay axis τ of the impulse response into equal time delay segments called excess delay bins.
- Excess delay is the relative delay of the *ith* multipath component as compared to the first arriving component and is given by τ_i.
- τ_0 represents the first arriving signal and $\tau_0 = 0$.
- The maximum excess delay of the channel is given by $N \Delta \tau$.

- $1/2\Delta\tau$ is the useful frequency span of the model. That is, the model may be used to analyze transmitted RF signals having bandwidths which are less than $1/2\Delta\tau$.
- The baseband impulse response of a multipath channel can be expressed as

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\exp[j(2\pi f_c \tau_i(t) + \phi_i(\tau - \tau_i(t))]\delta(\tau - \tau_i(t)) \qquad (5.12)$$

- The effect of spatial filtering can be include in (5.12).

  (the effect of angle of arrival of each multipath component).

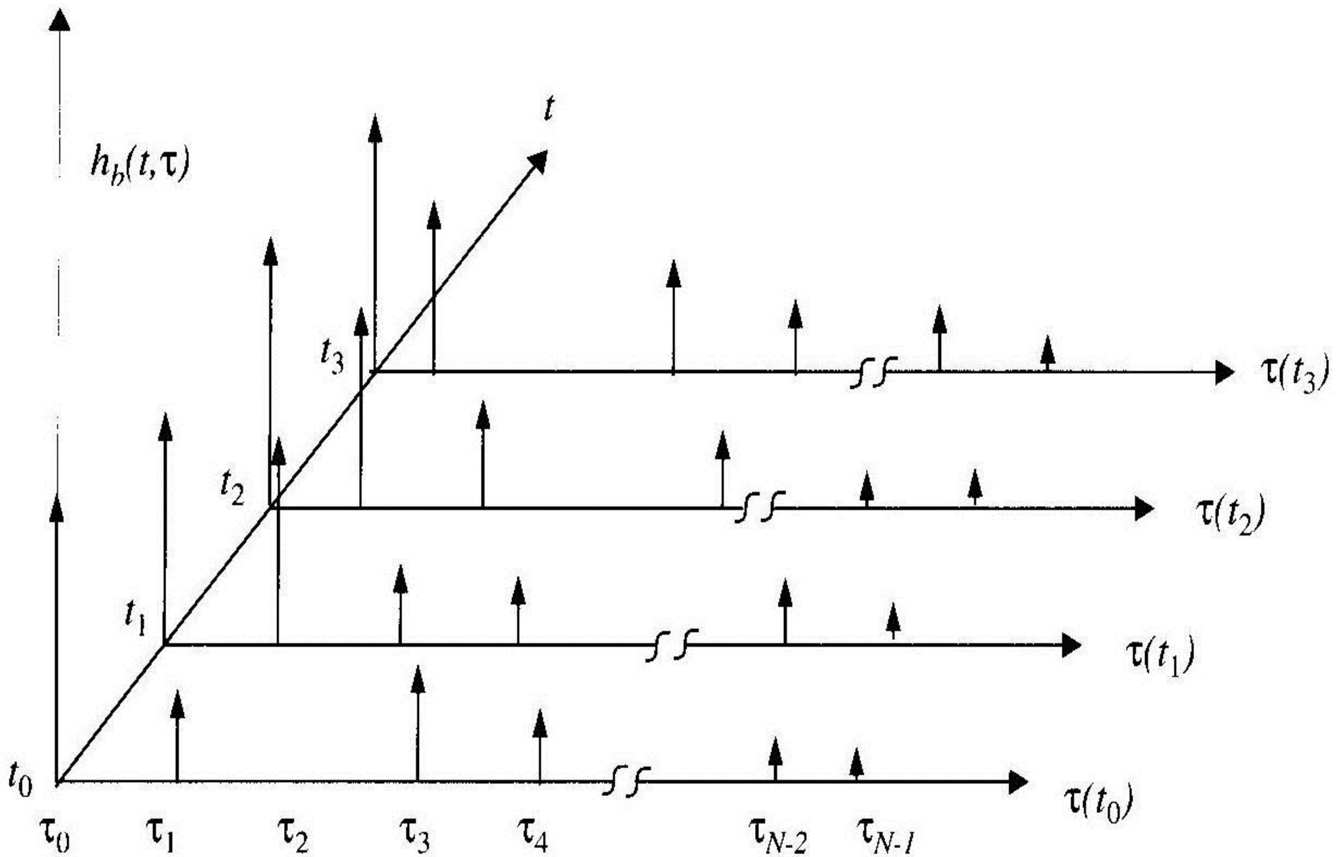


Figure 5.4 An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

- If there are two or more multipath signals that arrive within a excess delay bin, the multipath amplitude within an excess delay bin fade over a local area.

  If only one $\longrightarrow$ not fade significantly.

- If the channel is assumed to be time invariant,

$$h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp(j\theta_i)\delta(\tau - \tau_i)$$

- $P(t)$ is a probing signal that approximates a delta function.
- The power delay profile of the channel is found by taking the spatial average of $|h_b(t\,;\,\tau)|^2$.

5.2.1 Relationship Between Bandwidth and Receiver Power

- We now demonstrate how the small-scale fading behaves quite differently for two signals with different bandwidths in the identical multipath channel.
- Consider a pulse, transmitted RF signal of the form

$$x(t) = \mathrm{Re}\{p(t)\exp(j2\pi f_c t)\}$$

$$r(t) = \frac{1}{2}\sum_{i=0}^{N-1} a_i(\exp(j\theta_i))\cdot p(t-\tau_i)$$

$$= \sum_{i=0}^{N-1} a_i(\exp(j\theta_i))\cdot\sqrt{\frac{\tau_{\max}}{T_{bb}}}\, rect[\frac{t-\tau_i}{T_{bb}} - \frac{1}{2}]$$

$$|r(t_0)|^2 = \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} r(t) \times r^*(t) dt$$

$$= \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} \frac{1}{4} \mathrm{Re}\{\sum_{j=0}^{N-1} \sum_{i=0}^{N-1} a_j(t_0) a_i(t_0) p(t-\tau_j) p(t-\tau_i) \exp(j(\theta_j - \theta_i))\} dt$$

$$|r(t_0)|^2 = \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} \frac{1}{4} (\sum_{k=0}^{N-1} a_k^2(t_0) p^2(t-\tau_k)) dt$$

$$= \frac{1}{\tau_{\max}} \sum_{k=0}^{N-1} a_k^2(t_0) \int_0^{\tau_{\max}} \{\sqrt{\frac{\tau_{\max}}{T_{bb}}} rect[\frac{t-\tau_i}{T_{bb}} - \frac{1}{2}]\}^2 dt$$

$$= \sum_{k=0}^{N-1} a_k^2(t_0)$$

$$E_{a,\theta}[P_{WB}] = E_{a,\theta}[\sum_{i=0}^{N-1} |a_i \exp(j\theta_i)|^2] = \sum_{i=0}^{N-1} \overline{a_i^2}$$

- If a transmitted signal is able to resolve the multipaths, then the average small-scale received power is simply the sum of the average powers received in each multipath component.
- In practice, the amplitudes of individual multipath components do no fluctuate widely in local area.
- Consider a CW signal $c(t) = 2$.

$$r(t) = \sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))$$

$$|r(t)|^2 = |\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))|^2$$

- $r(t)$ has a large fluation in amplitude, because θ_i varies greatly.
- The average received power over a local area is given by

$$E_{a,\theta}[P_{CW}] = E_{a,\theta}[|\sum_{i=0}^{N-1} a_i \exp(j\theta_i)|^2]$$

$$E_{a,\theta}[P_{CW}] \approx \overline{[(a_0 e^{j\theta_0} + a_1 e^{j\theta_1} + \ldots + a_{N-1} e^{j\theta_{N-1}})}$$

$$\overline{\times (a_0 e^{-j\theta_0} + a_1 e^{-j\theta_1} + \ldots + a_{N-1} e^{-j\theta_{N-1}})}]$$

$$E_{a,\theta}[P_{CW}] \approx \sum_{i=0}^{N-1} \overline{a_i^2} + 2 \sum_{i=0}^{N-1} \sum_{i,j\neq 1}^{N} r_{i,j} \overline{\cos(\theta_i - \theta_j)}$$

$$r_{ij} = E_a[a_i a_j]$$

- Average power are the same in two cases; wideband and narrow band cases.

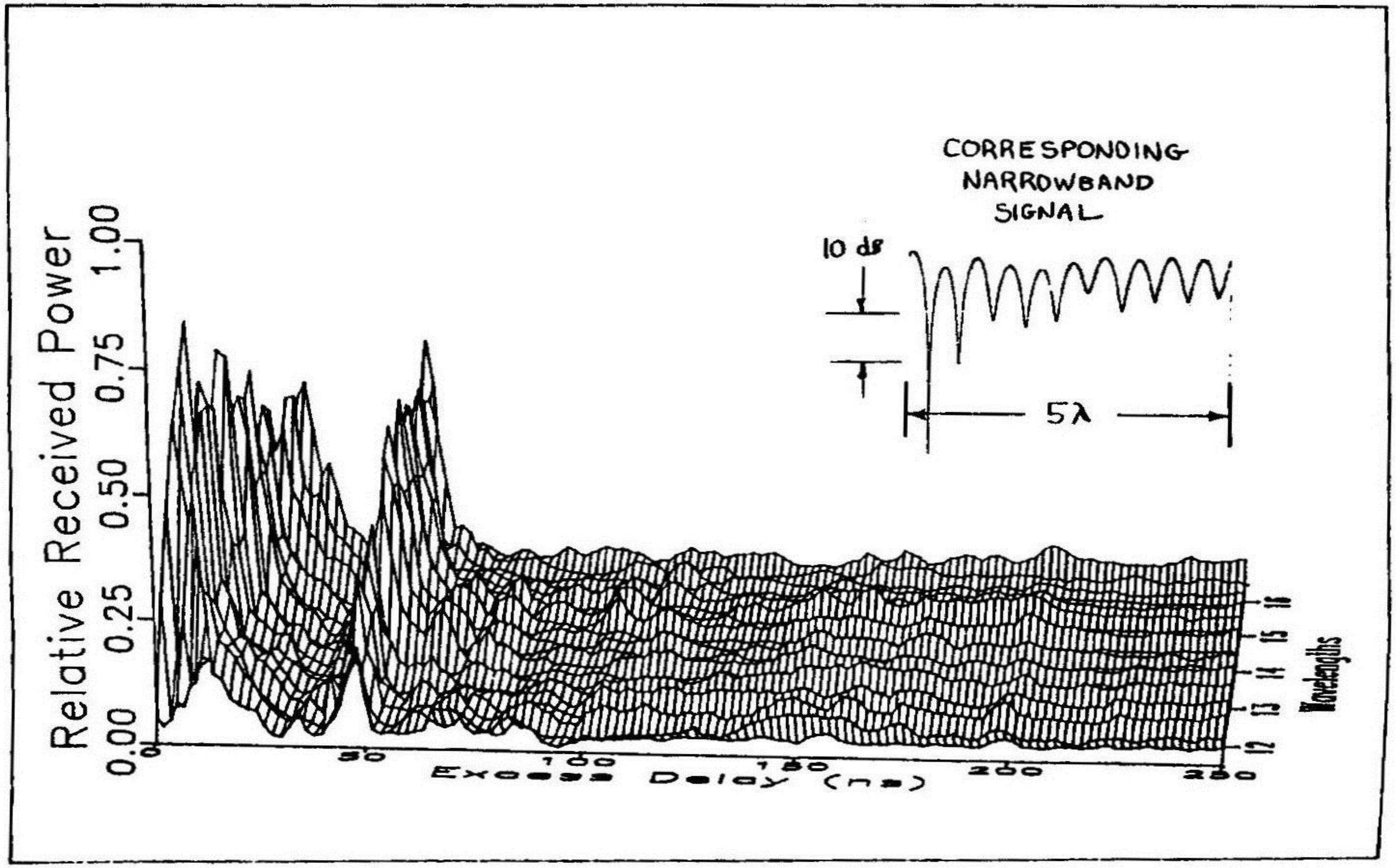


Figure 5.5 Measured wideband and narrowband received signals over a 5λ (0.375 m) measurement track inside a building. Carrier frequency is 4 GHz. Wideband power is computed using Equation (5.19), which can be thought of as the area under the power delay profile. The axis into the page is distance (wavelengths) instead of time.

Example 5.2

Assume a mobile traveling at a velocity of 10 m/s receives two multipath components at a carrier frequency of 1000 MHz. The first component is assumed to arrive at $\tau = 0$ with an initial phase of 0° and a power of -70 dBm, and the second component which is 3 dB weaker than the first component is assumed to arrive at $\tau = 1$ μs, also with initial phase of 0°. If the mobile moves directly toward the direction of arrival of the first component and directly away from the direction of arrival of the second component, compute the narrowband instantaneous power at time intervals of 0.1 s from 0s to 0.5 s. Compute the average narrowband power received over this observation interval. Compute average narrowband and wideband received powers over the interval, assuming the amplitudes of the two multipath components do not fade over the local area.

Solution:

$\tau_N = N\Delta\tau$, $\tau_N = 100\mu s$ and $N = 64 \Rightarrow \Delta\tau = \tau_N$ / N= 1.5625 μs.

2/ $\Delta\tau$ = 2 / 1.5625 = 1.28 MHz.

For the SMRCIM urban microcell model $\tau_N = 4\mu s$ and

$\Delta\tau = \tau_N$ / N= 62.5

2/ $\Delta\tau$ = 2 / 62.5 = 32 MHz.

Indoor channel,

$\Delta\tau = \tau_N$ / N= $\frac{500 \times 10^{-9}}{64}$ = 7.8125

2/ $\Delta\tau$ = 2 / 7.8125 = 256 MHz.

Example 5.3

Assume a discrete channel impulse response is used to model urban RF radio channels with excess delays as large as 100 μs. If the number of multipath bins is fixed at 64, find (a) $\Delta\tau$, and (b) the maximum RF bandwidth which the two models can accurately represent. Repeat the exercise for an indoor channel model with excess delays as large as 500 ns. As described in section 5.7.6, SMRCIM are statistical channel model based on Equation(5.12) that use parameters in this example.

Solution:

$$\lambda = \frac{c}{f} = \frac{3\times 10^8}{1000\times 10^6} = 0.3\text{m}$$, -70dB = 100 Pw.

t = 0, phase of both multipath component are 0°

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))\right|^2$$

$$= \left|\sqrt{100\text{pW}}\times\exp(0) + \sqrt{50\text{pW}}\times\exp(0)\right|^2 = 291\text{pW}$$

As the mobile moves, the phase of two multipath components changes is opposite directions.

t = 0.1, the first component

$$\theta_i = \frac{2\pi d}{\lambda} = \frac{2\pi vt}{\lambda} = \frac{2\pi\times 10(\text{m/s})\times 0.1}{0.3} = 120^\circ$$

Solution:

$t = 0.2,\ \theta_1 = 120°,\ \theta_2 = -120°$

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))\right|^2$$

$$= \left|\sqrt{100\text{pW}} \times \exp(j120^\circ) + \sqrt{50\text{pW}} \times \exp(-j120^\circ)\right|^2 = 79.3\text{pW}$$

$t = 0.3,\ \theta_1 = 360°,\ \theta_2 = -360°$

$$|r(t)|^2 = \left|\sum_{i=0}^{N-1} a_i \exp(j\theta_i(t,\tau))\right|^2$$

$$= \left|\sqrt{100\text{pW}} \times \exp(j0^\circ) + \sqrt{50\text{pW}} \times \exp(-j0^\circ)\right|^2 = 291\text{pW}$$

$t = 0.4$, $|r(t)|^2 = 79.3$pW, t = 0.5, $|r(t)|^2 = 79.3$pW.

The average of narrowband received power is

$$\frac{2(291) + 4(7.93)}{6} = 149\,\text{pW}$$

Solution:

$$E_{a,\theta}[P_{W,B}] = E_{a,\theta}[|\sum_{i=0}^{N-1} a_i \exp(j\theta_i)|^2] \approx \sum_{i=0}^{N-1} \overline{a_i^2}$$

$$E_{a,\theta}[P_{W,B}] = 100 + 50 = 150\text{pW}$$

5.3 Small – Scale Multipath Measurements

5.3.1 Direct RF Pulse System

- This system transmits a repetitive pulse of width T_{bb}s, and uses a receiver with a wide bandpass filter$\left(BW = 2/T_{bb}\text{Hz}\right)$.

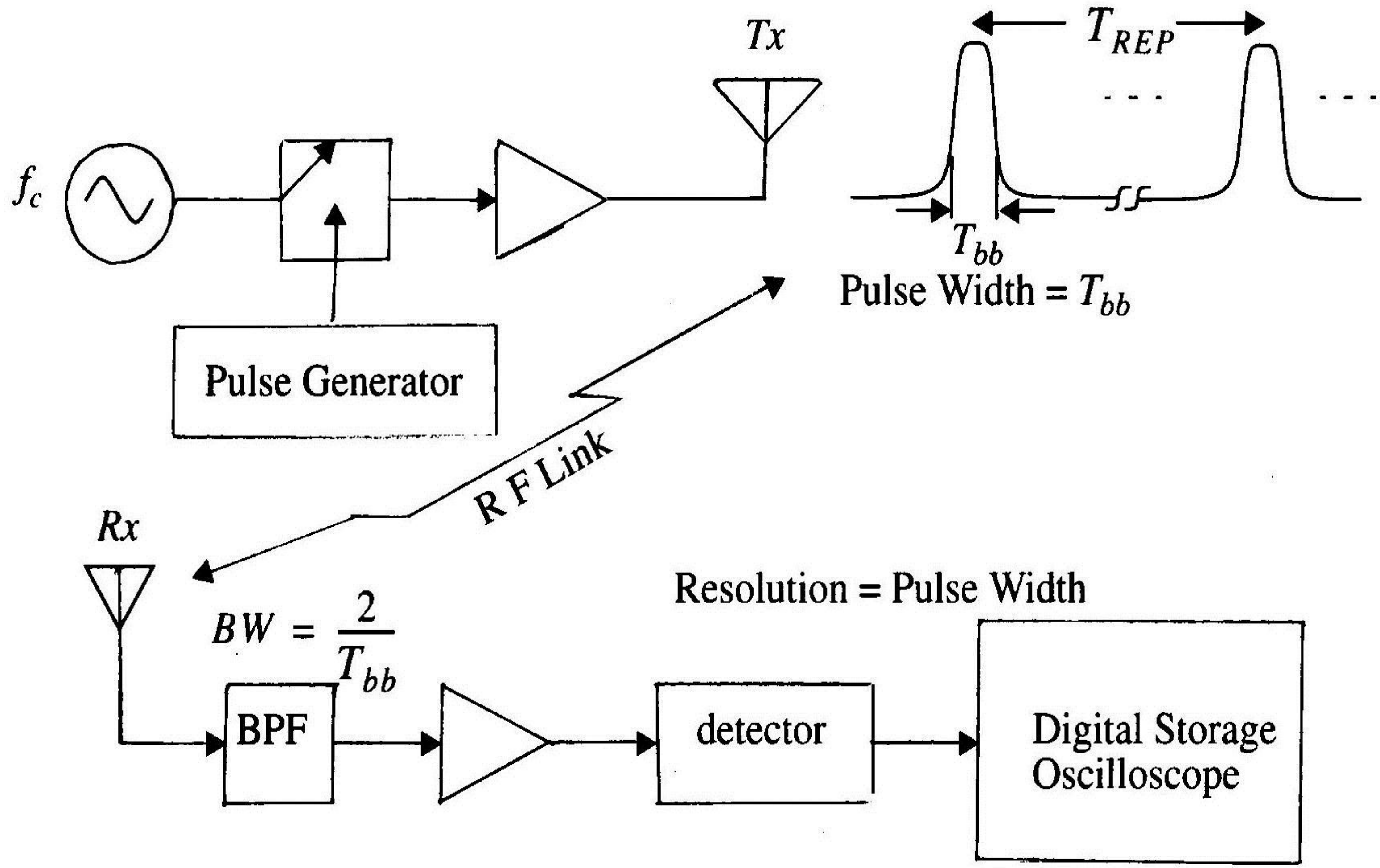


Figure 5.6 Direct RF channel impulse response measurement system.

- This gives an immediate measurement of the square of the channel impulse response convolved with the probing pulse.
- This system can also provide a local average power delay profile.
- The lack of complexity is its advantage.

- The minimum resolvable delay between multipath component is equal to the probing pulse T_{bb}.
- Disadvantage :
  1) subject to interference and noise.
  2) oscilloscope may not trigger properly.
  3) the phase of the individual multipath components are not received. (can use a coherent detector to solve this problem)

5.3.2 Spread Spectrum Sliding Correlator Channel Sounding

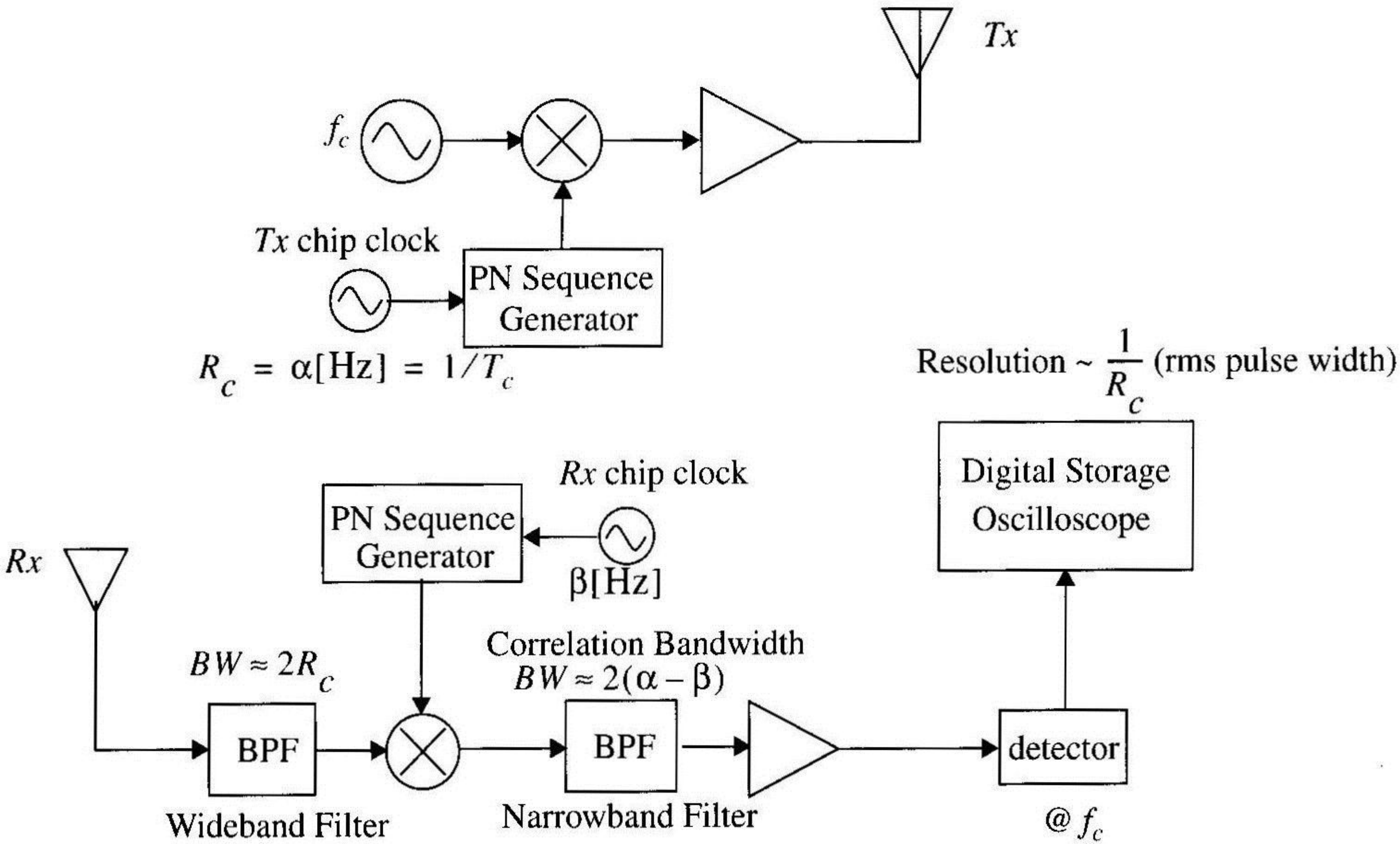


Figure 5.7 Spread spectrum channel impulse response measurement system.

- The advantage of a spread spectrum system is that, while the probing signal may be wideband, it is possible to detect the transmitted signal using a narrowband receiver.
- Spread spectrum systems : pseudo-noise (PN) sequence, chip duration T_c , chip rate $R_c = \frac{1}{T_c} H_8$

$$BW = 2R_c$$

- A sliding correlator : the transmitter chip clock is run at a slightly faster rate than the receiver chip clock.
- The narrowband filter that follows the correlator can reject almost all of the incoming signal power, when the two sequences are not identically aligned.

- Processing gain :

$$PG = \frac{2R_c}{R_{bb}} = \frac{2T_{bb}}{T_c}$$

where $T_{bb} = 1/R_{bb}$ is the period of the baseband information.

In this case, $R_{bb} = (\alpha - \beta)$.

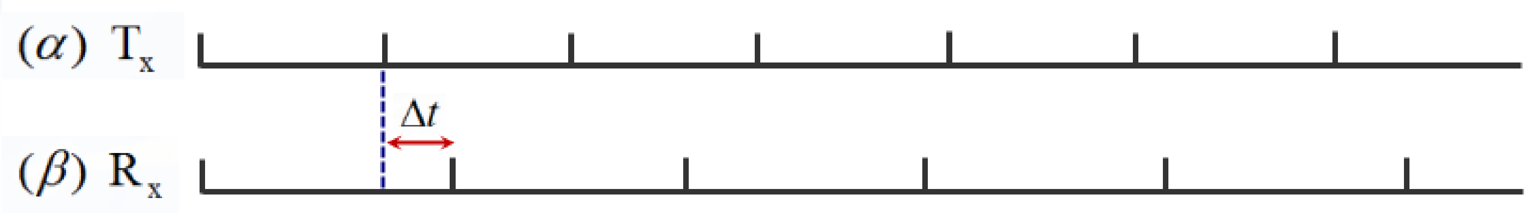
- After envelope detection, the channel impulse response convolved with the pulse shape of a single chip is displayed on the oscilloscope.
- The time resolution $(\Delta\tau)$ of multipath components is

$$\Delta\tau = 2T_c = \frac{2}{R_c}$$

In actuality, is on the order of T_c.

- The time between maximum correlations (ΔT) can be calculated from Equation 5.30

$$\Delta T = T_c \gamma l = \frac{\gamma l}{R_c}$$



$$\Delta t = \frac{l}{\beta} - \frac{l}{\alpha} = l\,\frac{\alpha - \beta}{\alpha\beta} = \frac{l}{\beta} \times \frac{1}{\gamma}$$

- where γ , slide factor, $= \frac{\alpha}{\alpha\beta}$

  $l = 2^n - 1$, sequence length.

$$\Delta T = \left(\frac{l}{\alpha}\right) \times \frac{1}{\Delta t} \times \left(\frac{l}{\alpha}\right) = \frac{l^2}{\alpha^2} \cdot \frac{\beta\gamma}{l} = \frac{l\gamma}{\alpha} = \frac{\gamma l}{R_c}$$

- $\text{Actual Propagation Time} = \dfrac{\text{observed time}}{\gamma}.$

  This effect is known as time dilation.
- Caution must be taken to ensure that the sequence length has a period which is greater than the longest multipath propagation delay. The sequence period is

$$\tau_{PNseq} = T_c l$$

- Advantages :
  1) reject passband noise, this improving the coverage range.
  2) eliminate synchronization
  3) require lower transmitter power
  4) sensitivity is adjustable.

- Disadvantages :
  1) not made in real time
  2) the time required is longer
  3) phases of individual multipath components can not be measured, even if coherent detector is used.

5.3.3 Frequency Domain Channel Sounding

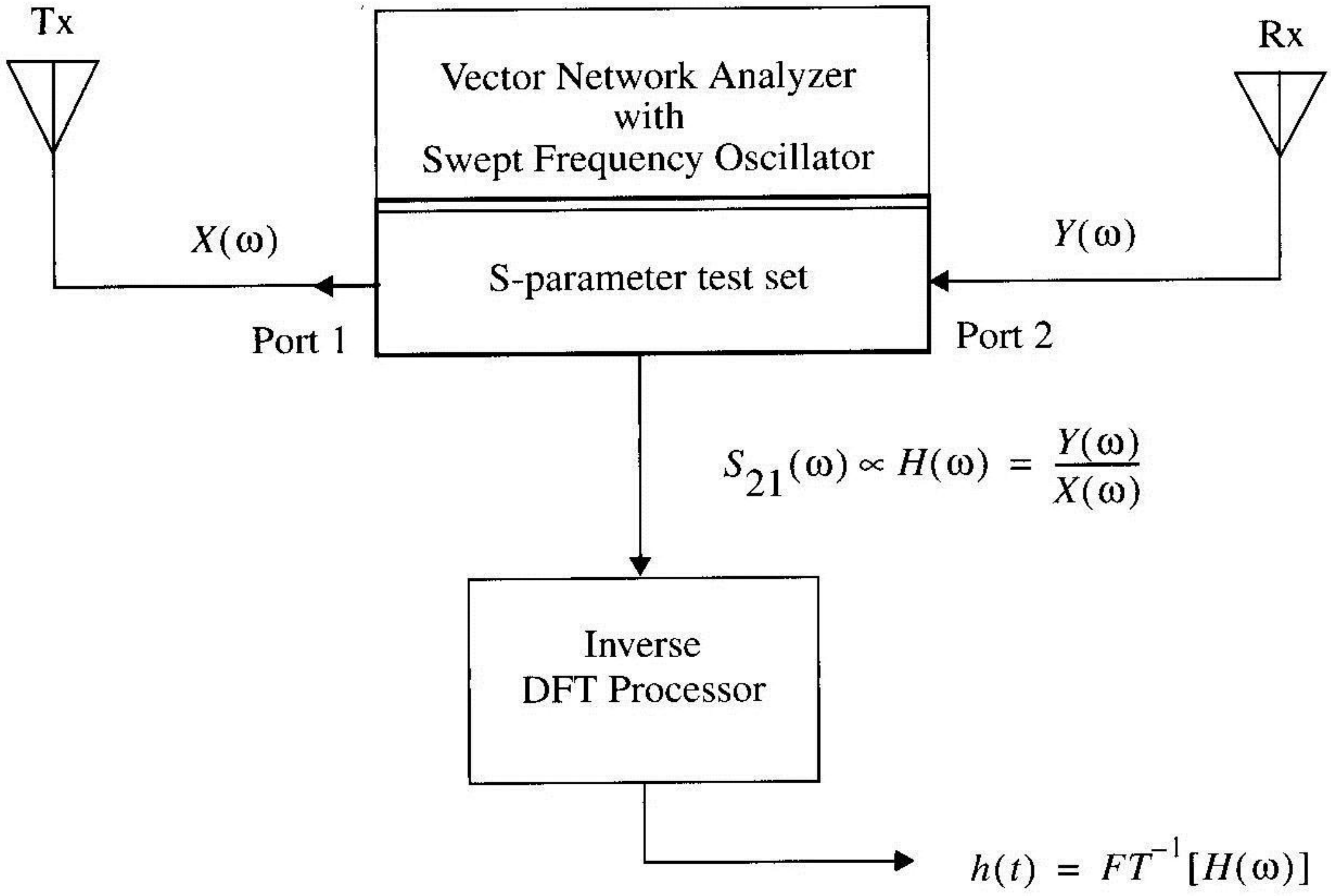


Figure 5.8 Frequency domain channel impulse response measurement system.

- Measure the channel impulse response in the frequency domain.
- In theory, this technique works. However,
  1) the system requires careful calibration and hardwired synchronization between *Tx* and *Rx*. (only for very close measurements, e.g., indoor channel sounding)
  2) the non-real-time nature. For time varying channels, fast sweep times are necessary.

5.4 Parameters of Mobile Multipath Channaels

- Many multipath channel parameters are derived from the power delay profile.
- Researchers often to choose to sample at spatial separations of a quarter of a wave length and over receiver movements no greater than 6m in outdoor channels and no greater than 2m indoor channels in the 450 MHz – 6 GHz range.

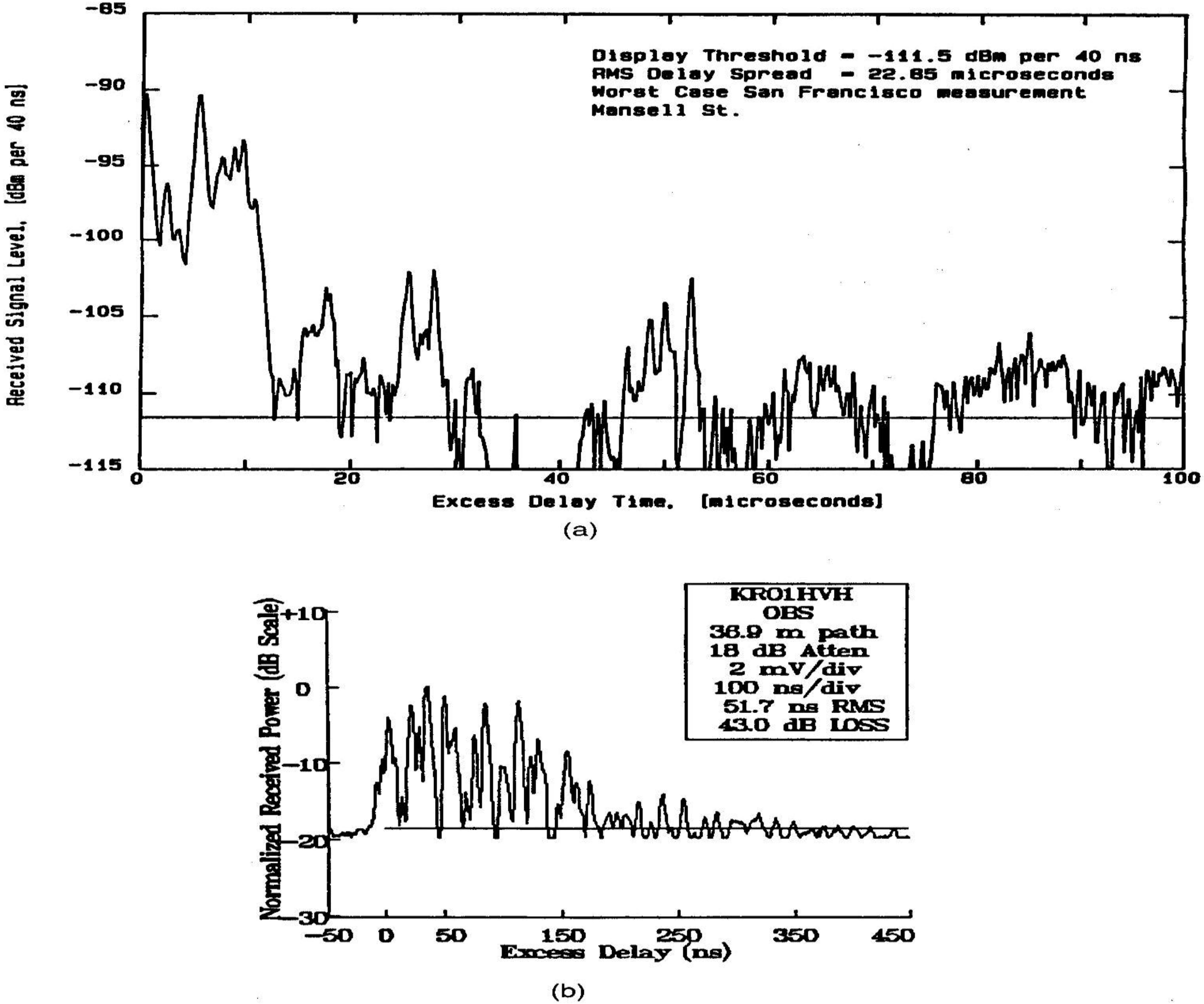


Figure 5.9 Measured multipath power delay profiles: a) From a 900 MHz cellular system in San Francisco [from [Rap90] © IEEE]; b) inside a grocery store at 4 GHz [from [Haw91] © IEEE].

5.4.1 Time Dispersion Parameters

- The mean excess delay, rms delay spread, and excess delay spread (X dB) are multipath channel parameters that can be determined from a power delay profile.
- The mean excess delay –

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- The rms delay spread –

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

- These delays are measured relative to the first detectable signal arriving at the receiver at $\tau_0 = 0$
- Typical values of rms delay spread are on the order of microseconds in outdoor – nanoseconds in indoor.

Table 5.1 Typical Measured Values of RMS Delay Spread

Environment	Frequency (MHz)	RMS Delay Spread (σ_τ)	Notes	Reference
Urban	910	1300 ns avg. 600 ns st. dev. 3500 ns max.	New York City	[Cox75]
Urban	892	10–25 μs	Worst case San Francisco	[Rap90]
Suburban	910	200–310 ns	Averaged typical case	[Cox72]
Suburban	910	1960–2110 ns	Averaged extreme case	[Cox72]
Indoor	1500	10–50 ns 25 ns median	Office building	[Sal87]
Indoor	850	270 ns max.	Office building	[Dev90a]
Indoor	1900	70–94 ns avg. 1470 ns max.	Three San Francisco buildings	[Sei92a]

- The maximum excess delay (X dB) – the time delay during which multipath energy falls to X dB below the maximum.

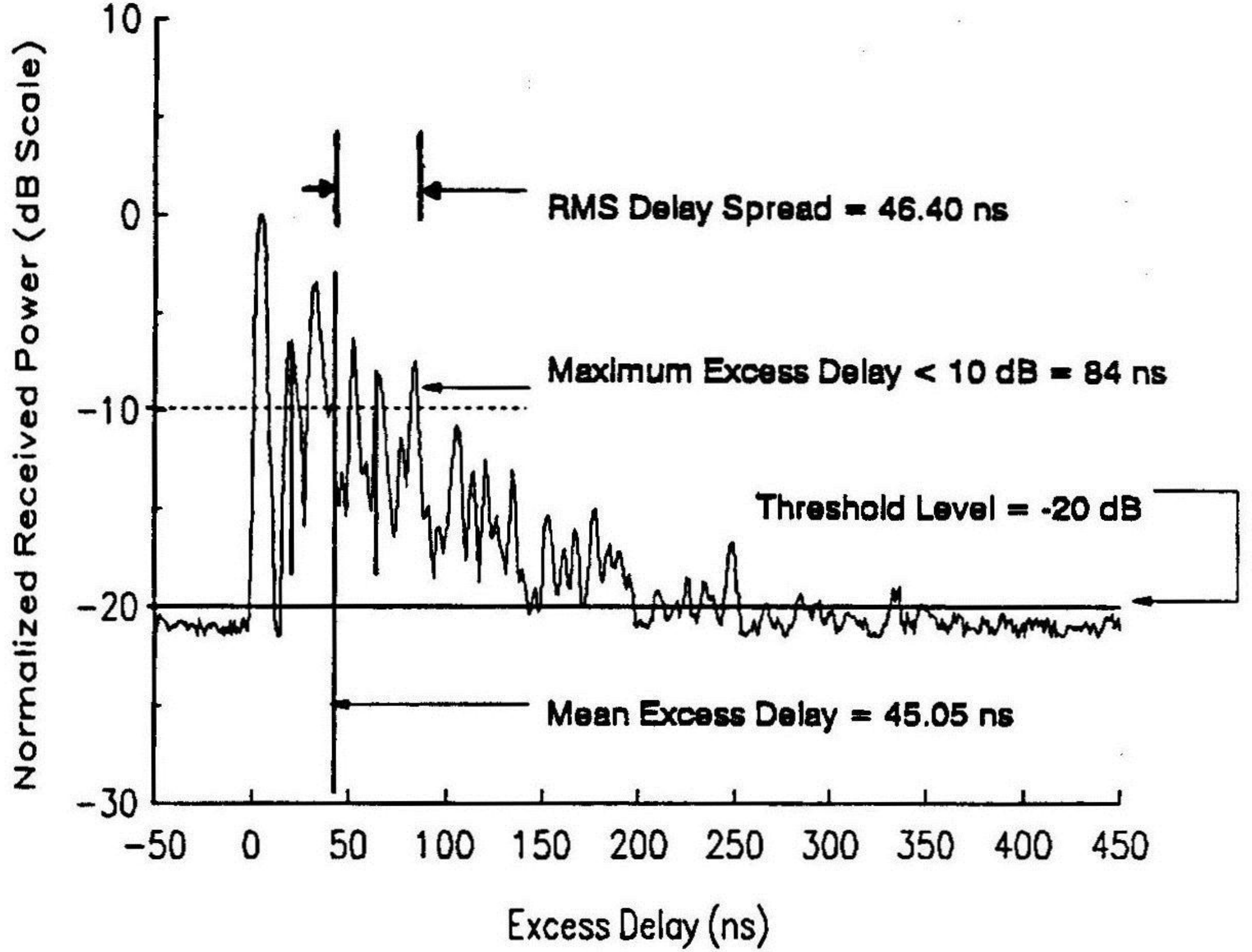


Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

- The value of τ_x is sometimes called the excess delay spread of a power delay profile.

5.4.2 Coherence Bandwidth

- Coherence bandwidth is a statistical measure of the range of frequencies over which the channel can be considered "flat".
- The coherence bandwidth B_c is a defined relation derived from the rms delay spread.

- If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then the coherence bandwidth is approximately

$$B_c \approx \frac{1}{50\sigma_\tau}$$

- If correlation function is above 0.5,

$$B_c \approx \frac{1}{5\sigma_\tau}$$

- It is important to note that an exact relationship between coherence bandwidth and rms delay spread is a function of specific channel impulse responses and applied signals.

Example 5.5

Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel. Would this channel be suitable for AMPS or GSM service without the use of an equalizer?

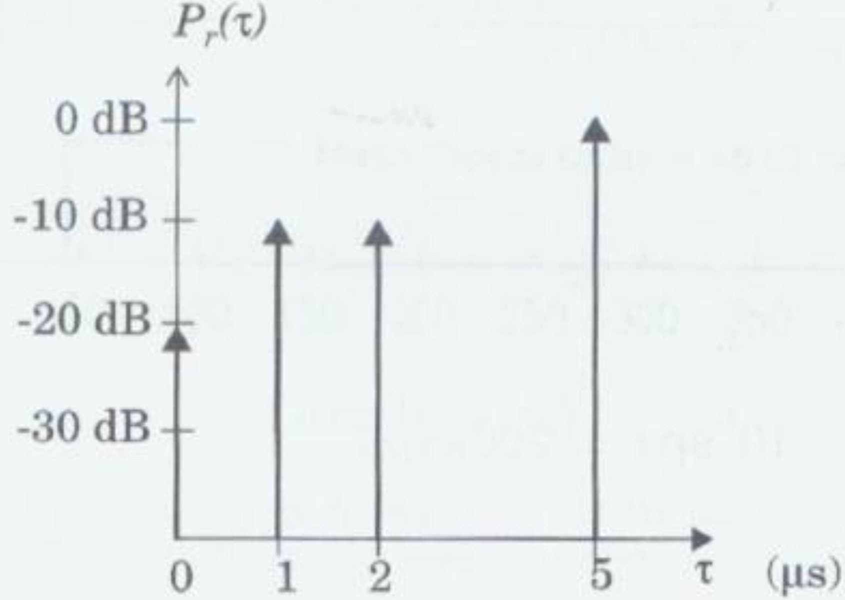


Figure E5.5

Solution

Using the definition of maximum excess delay (10 dB), it can be seen that $\tau_{10\,dB}$ is 5 μs. The rms delay spread for the given multipath profile can be obtained using Equations (5.35)–(5.37). The delays of each profile are measured relative to the first detectable signal. The mean excess delay for the given profile is

$$\bar{\tau} = \frac{(1)(5) + (0.1)(1) + (0.1)(2) + (0.01)(0)}{[0.01 + 0.1 + 0.1 + 1]} = 4.38\ \mu s$$

The second moment for the given power delay profile can be calculated as

$$\overline{\tau^2} = \frac{(1)(5)^2 + (0.1)(1)^2 + (0.1)(2)^2 + (0.01)(0)}{1.21} = 21.07\ \mu s^2$$

Therefore the rms delay spread is $\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\ \mu s$

The coherence bandwidth is found from Equation (5.39) to be

$$B_c \approx \frac{1}{5\sigma_\tau} = \frac{1}{5(1.37\mu s)} = 146\ \text{kHz}$$

Since B_c is greater than 30 kHz, AMPS will work without an equalizer. However, GSM requires 200 kHz bandwidth which exceeds B_c, thus an equalizer would be needed for this channel.

5.4.3 Doppler Spread and Coherence Time

- Delay spread and coherence bandwidth are parameters which describe the time dispersive nature of the channel.
- However, they do not offer information about the time varying nature of the channel.

- Doppler spread B_D is a measure of the spectral broadening caused by the time rate of change of the mobile radio channel.
- Doppler spread is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero.
- If the baseband signal bandwidth is much greater than ,the effects of Doppler spread are negligible at the receiver – slow fading channel.

- Coherence time T_c - a statistical measure of the time duration over which the channel impulse response is essentially invariant.
- Coherence time is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain.

- If the time correlation function is above 0.5,

$$T_c \approx \frac{9}{16\pi f_m}$$

f_m is the maximum Doppler shift given by $f_m = \frac{v}{\lambda}$

- A more popular definition for modern digital communications is

$$T_c = \sqrt{\frac{9}{16\pi f_m{}^2}} = \frac{0.423}{f_m}$$

Example 5.6

Determine the proper spatial sampling interval required to make small-scale propagation measurements which assume that consecutive samples are highly correlated in time. How many samples will be required over 10 m travel distance if $f_c = 1900$ MHz and $v = 50$ m/s. How long would it take to make these measurements, assuming they could be made in real time from a moving vehicle? What is the Doppler spread B_D for the channel?

Solution

For correlation, ensure that the time between samples is equal to $T_C/2$, and use the smallest value of T_C for conservative design.

Using Equation (5.40.b)

$$T_C \approx \frac{9}{16\pi f_m} = \frac{9\lambda}{16\pi v} = \frac{9c}{16\pi v f_c} = \frac{9 \times 3 \times 10^8}{16 \times 3.14 \times 50 \times 1900 \times 10^6}$$

$$T_C = 565\mu s$$

Taking time samples at less than half T_C, at 282.5 μs corresponds to a spatial sampling interval of

$$\Delta x = \frac{vT_C}{2} = \frac{50 \times 565\,\mu s}{2} = 0.014125\text{ m} = 1.41\text{ cm}$$

Therefore, the number of samples required over a 10 m travel distance is

$$N_x = \frac{10}{\Delta x} = \frac{10}{0.014125} = 708\text{ samples}$$

The time taken to make this measurement is equal to $\frac{10\text{ m}}{50\text{ m/s}} = 0.2\text{ s}$

The Doppler spread is $B_D = f_m = \frac{vf_c}{c} = \frac{50 \times 1900 \times 10^6}{3 \times 10^8} = 316.66\text{ Hz}$

5.5 Types of Small – Scale Fading

- While multipath delay spread leads to time dispersion and frequency selective fading, Doppler spread leads to frequency dispersion and time selective fading.

Small-Scale Fading
(Based on multipath time delay spread)

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

5.5.1 Fading Effects Due to Multipath Times Delay Spread

- Flat fading

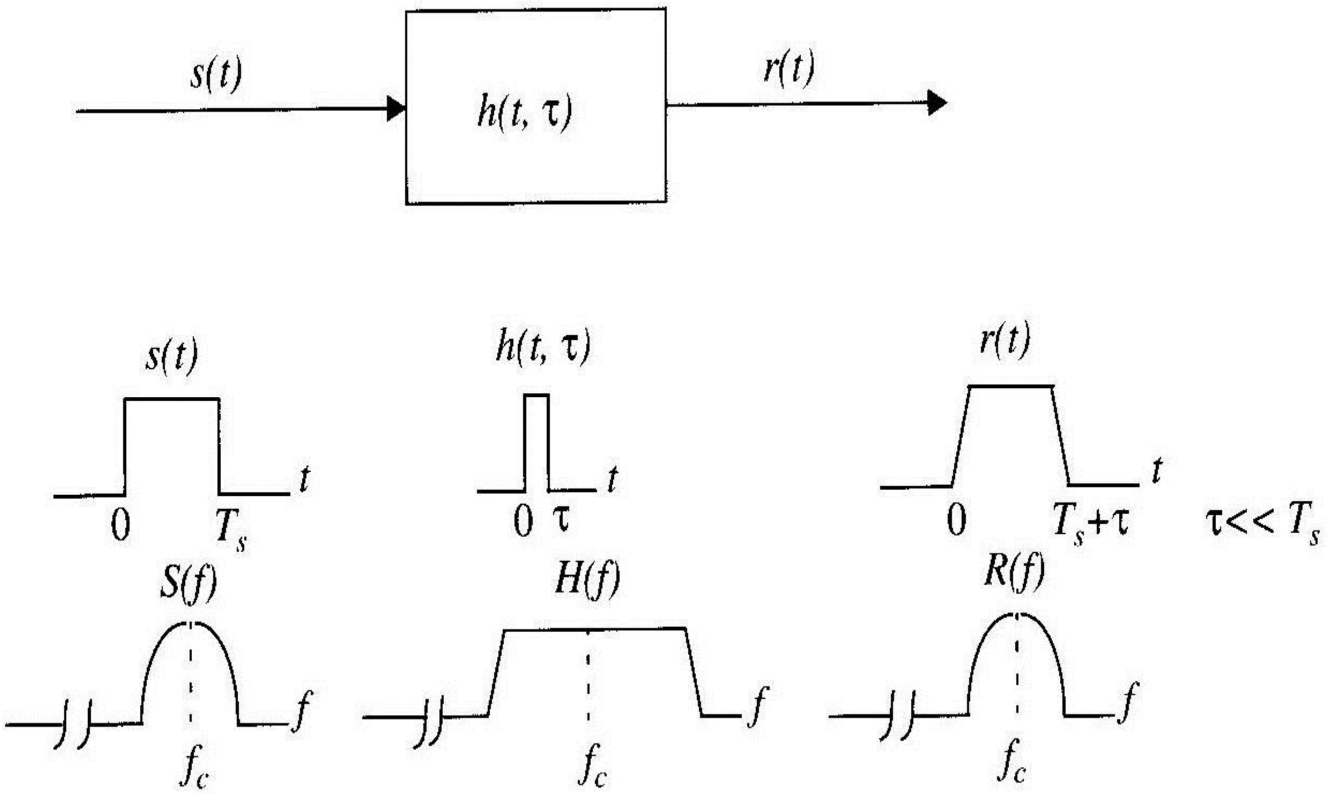


Figure 5.12 Flat fading channel characteristics.

- Flat fading channels are also known as amplitude varying channels and are sometimes referred to as narrowband channels, since the bandwidth of the applied signal is narrow as compared to the channel flat fading bandwidth.
- The most common amplitude distribution is the Rayleigh distribution.
- $B_S << B_C$ and $T_S >> \sigma_\tau$

5.5.1.2 Frequency Selective Fading

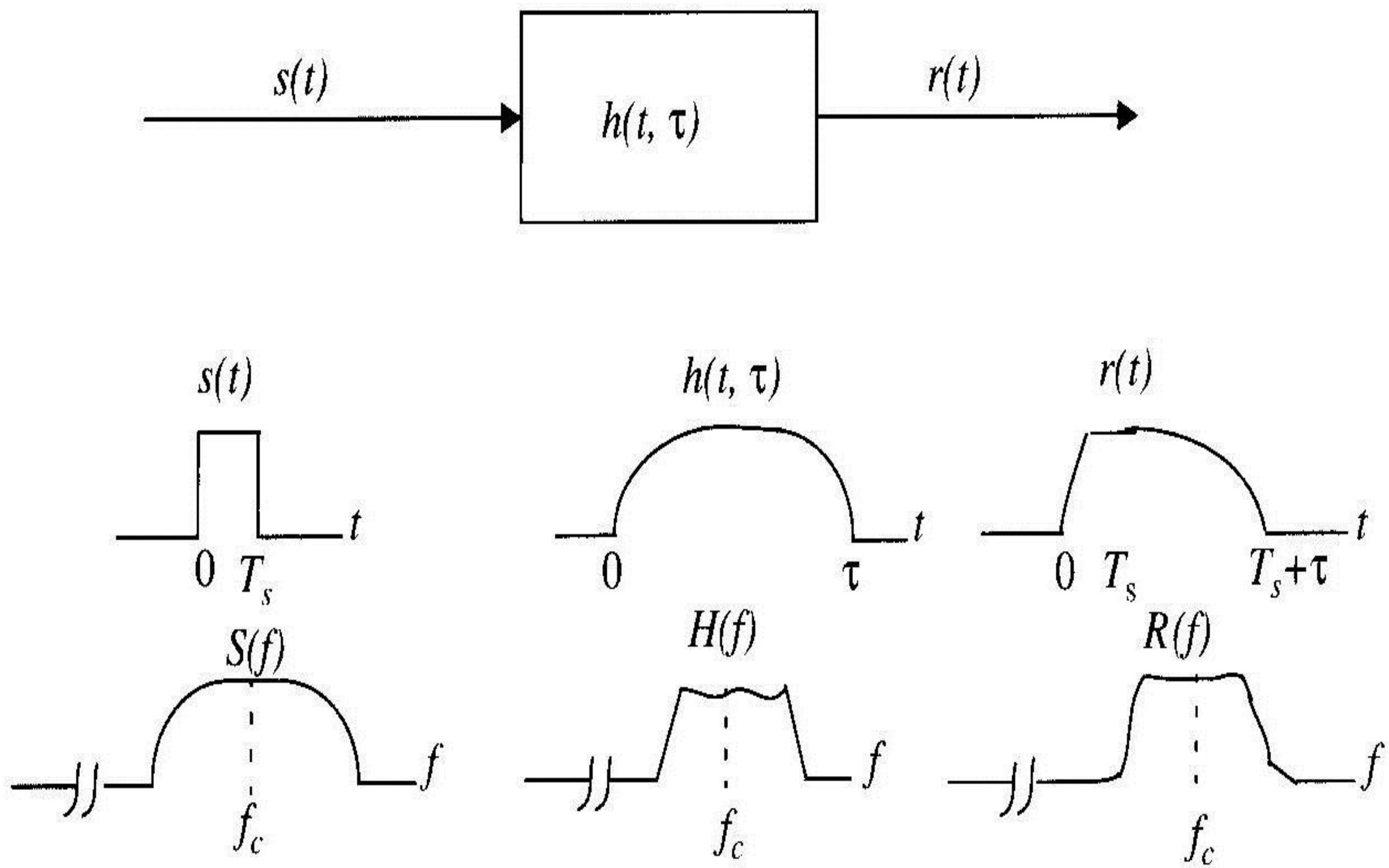


Figure 5.13 Frequency selective fading channel characteristics.

- inter-symbol interference (ISI)
- The two-ray Rayleigh fading model are generally used for analyzing frequency selective small-scale fading.
- $B_S > B_C$ and $T_S < \sigma_\tau$

5.5.2 Fading Effects Due to Doppler Spread

- Fast Fading – $T_S > T_C$ and $B_S < B_D$
- In practice, fast fading only occurs for very low data rates.
- Slow Fading – $T_S << T_C$ and $B_S >> B_D$

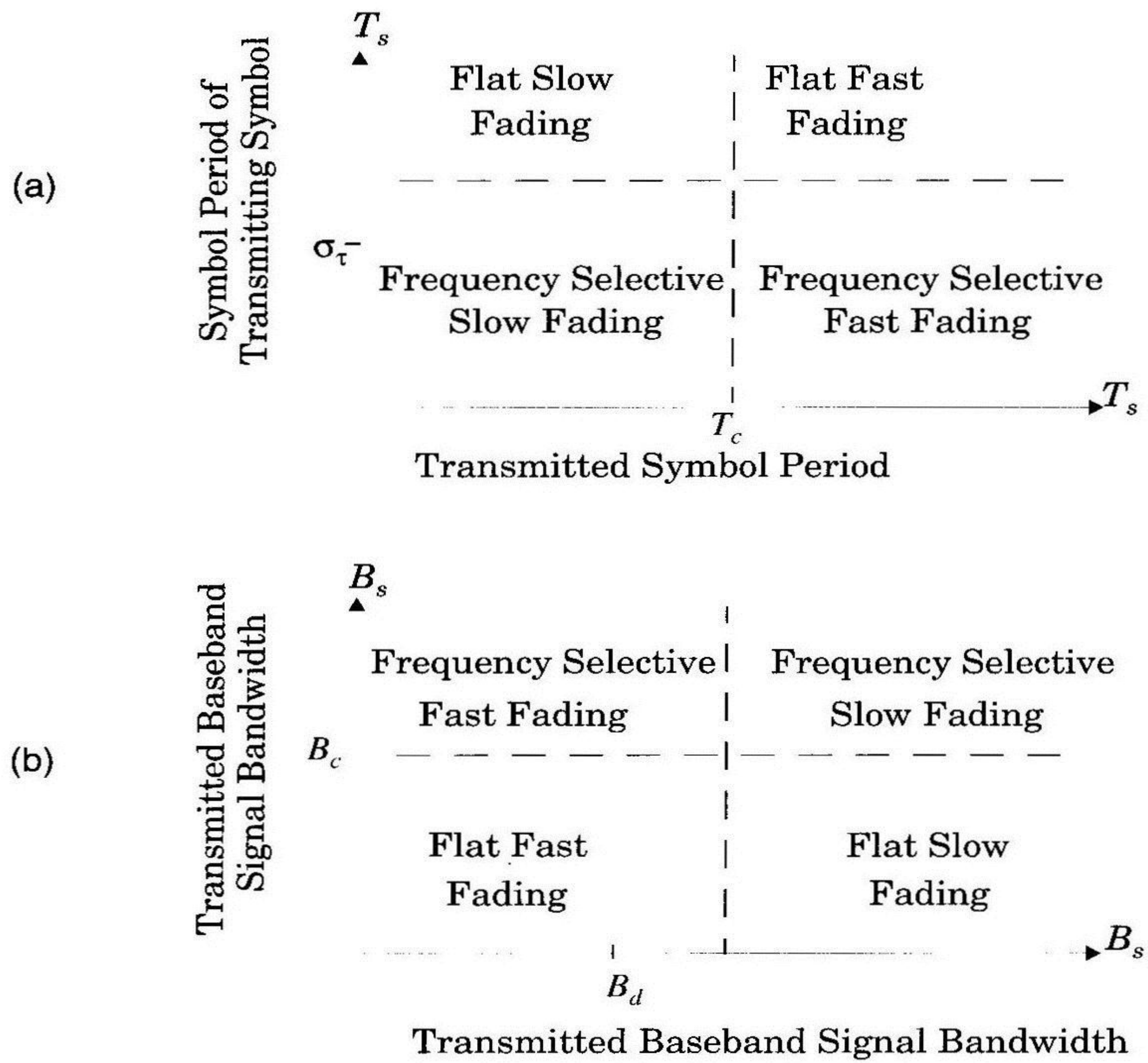


Figure 5.14 Matrix illustrating type of fading experienced by a signal as a function of: (a) symbol period; and (b) baseband signal bandwidth.

5.6 Rayleigh and Ricean Distributions

5.6.1 Rayleigh Fading Distribution

- It is well known that the envelope of the sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution.

$$p(r) = \begin{cases} \dfrac{r}{\sigma^2} \exp\left(-\dfrac{r^2}{2\sigma^2} \right) & (0 \le \mathrm{r} \le \infty) \\ 0 & (\mathrm{r} < 0) \end{cases}$$

where σ is the rms value of the received voltage signal before envelope detection, and σ^2 is the time-average power of the received signal before envelope detection.

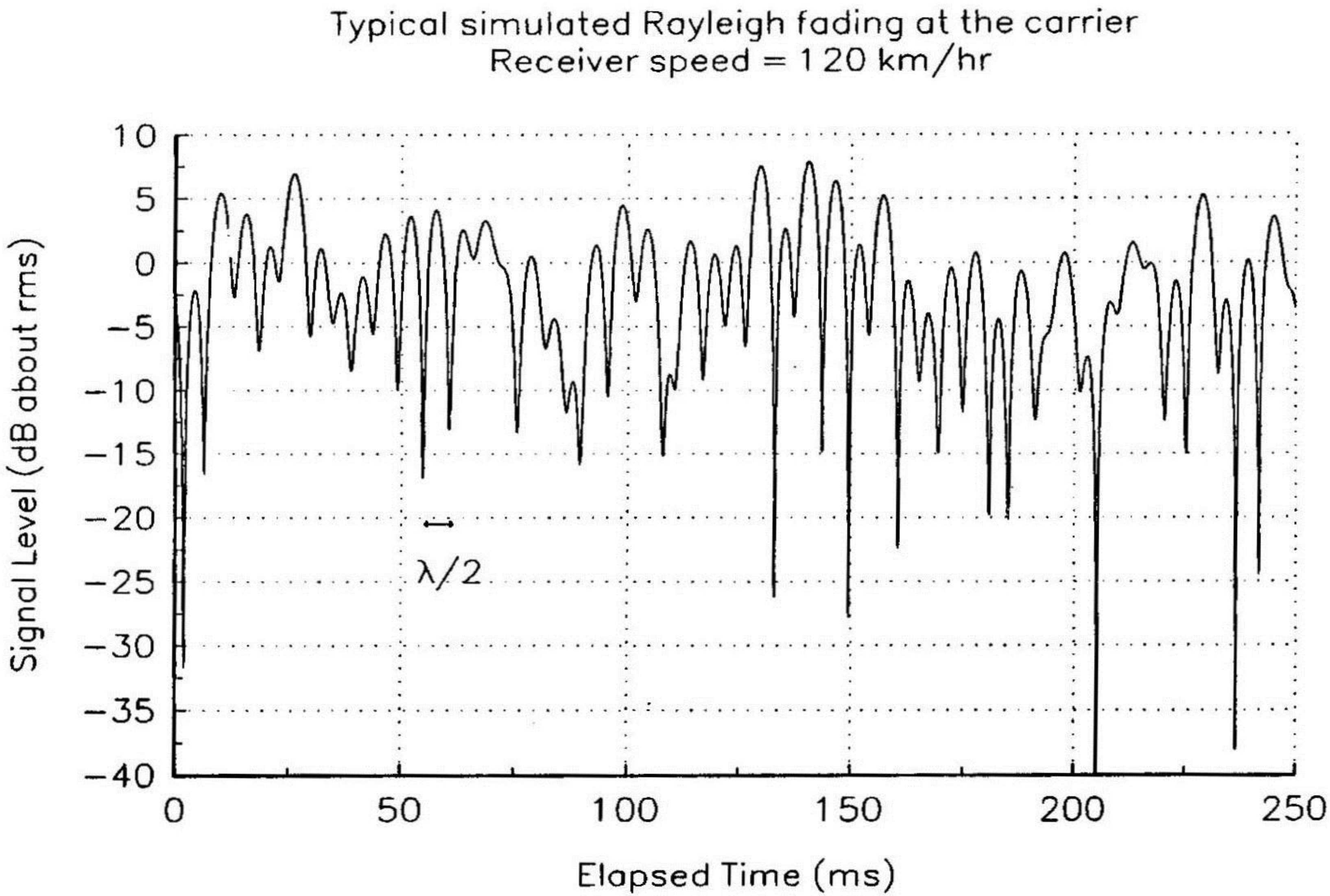


Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

- The corresponding cumulative distribution function (CDF)

$$P(R) = Pr(r \le R) = \int_0^R p(r)dr = 1 - \exp\left(-\frac{R^2}{2\sigma^2}\right)$$

- The mean value is given by

$$r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

- and the variance is

$$\sigma_r^2 = E[r^2] - E^2[r] = \int_0^\infty r^2 p(r)dr - \frac{\sigma^2 \pi}{2}$$

$$= \sigma^2 \left(2 - \frac{\pi}{2} \right) = 0.4292\sigma^2$$

- The rms value of the envelope is the square root of the mean square, or $\sqrt{2}\sigma$.

- The median value of *r* is found by solving

$$\frac{1}{2} = \int_0^{r_{median}} p(r)dr$$

and is

$$r_{median} = 1.177\sigma$$

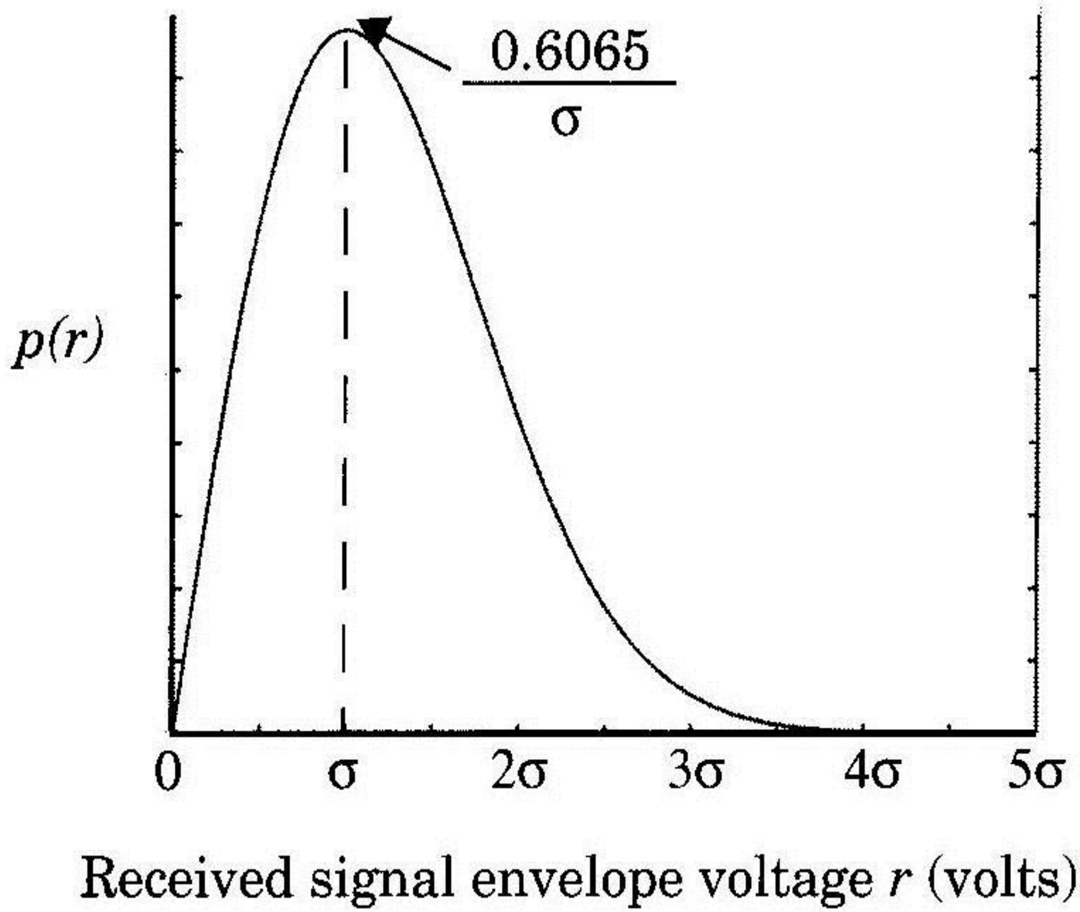


Figure 5.16 Rayleigh probability density function (pdf).

5.6.2 Ricean Fading Distribution

- When there is a dominant stationary (non-fading) signal component present, such as a line-of-sight propagation path, the small-scale fading envelope distribution is Ricean.

- The Ricean distribution is given by

$$p(r)=\begin{cases}\dfrac{r}{\sigma^2}e^{-\frac{(r^2+A^2)}{2\sigma^2}}I_0\left(\dfrac{Ar}{\sigma^2}\right) & \text{for}(A\geq 0,\mathrm{r}\geq 0)\\ 0 & \text{for}(\mathrm{r}<0)\end{cases}$$

The parameter *A* denotes the peak amplitude of the dominant signal and $I_0(\cdot)$ is the modified Bessel function of the first kind and zero-order.

- The Ricean distribution is often described in terms of a parameter *k* which is defined as

$$k = A^2 / (2\sigma^2)$$

or, in terms of dB

$$K(\text{dB}) = 10 \log \frac{A^2}{2\sigma^2} \text{ dB}$$

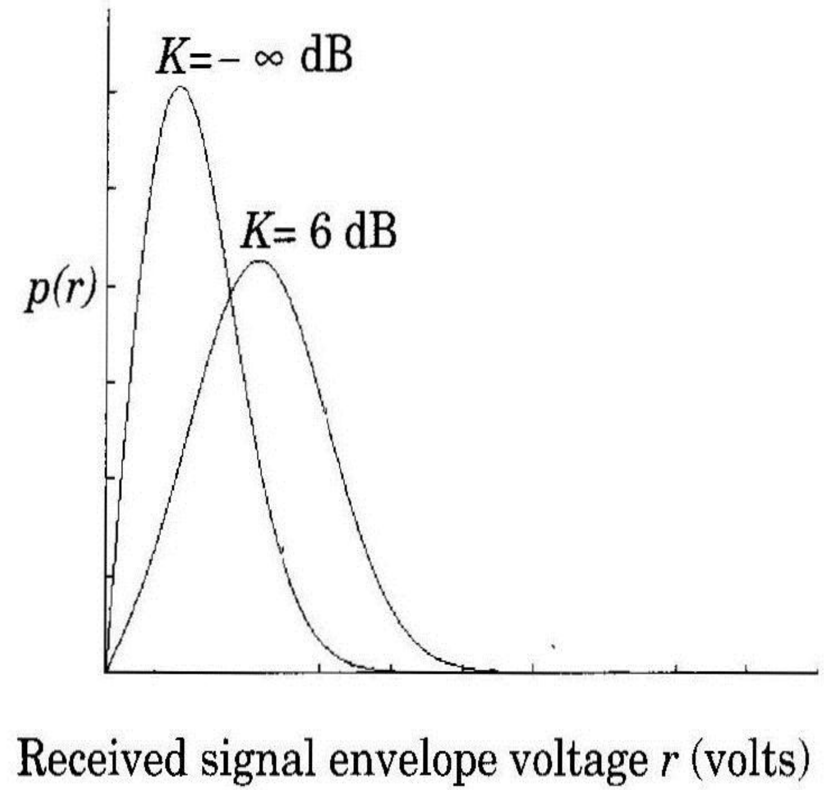


Figure 5.18 Probability density function of Ricean distributions: $K = -\infty$ dB (Rayleigh) and $K = 6$ dB. For K >> 1, the Ricean pdf is approximately Gaussian about the mean.

- As $A \rightarrow 0$, $k \rightarrow -\infty \text{dB}$, and as the dominant path decreases in amplitude, the Ricean distribution degenerates to a Rayleigh distribution.

5.7 Statistical Models for Multipath Fading Channels

5.7.1 Clark's Model for Flat Fading

- Clark's Model are deduced form scattering.
- The model assumes a fixed transmitter with a vertically polarized antenna.
- The field incident on the mobile antenna is assumed to be comprised of N azimuthal plane waves with arbitrary carrier phases, arbitrary azimuthal angles of arrival, and each wave having equal average amplitude.

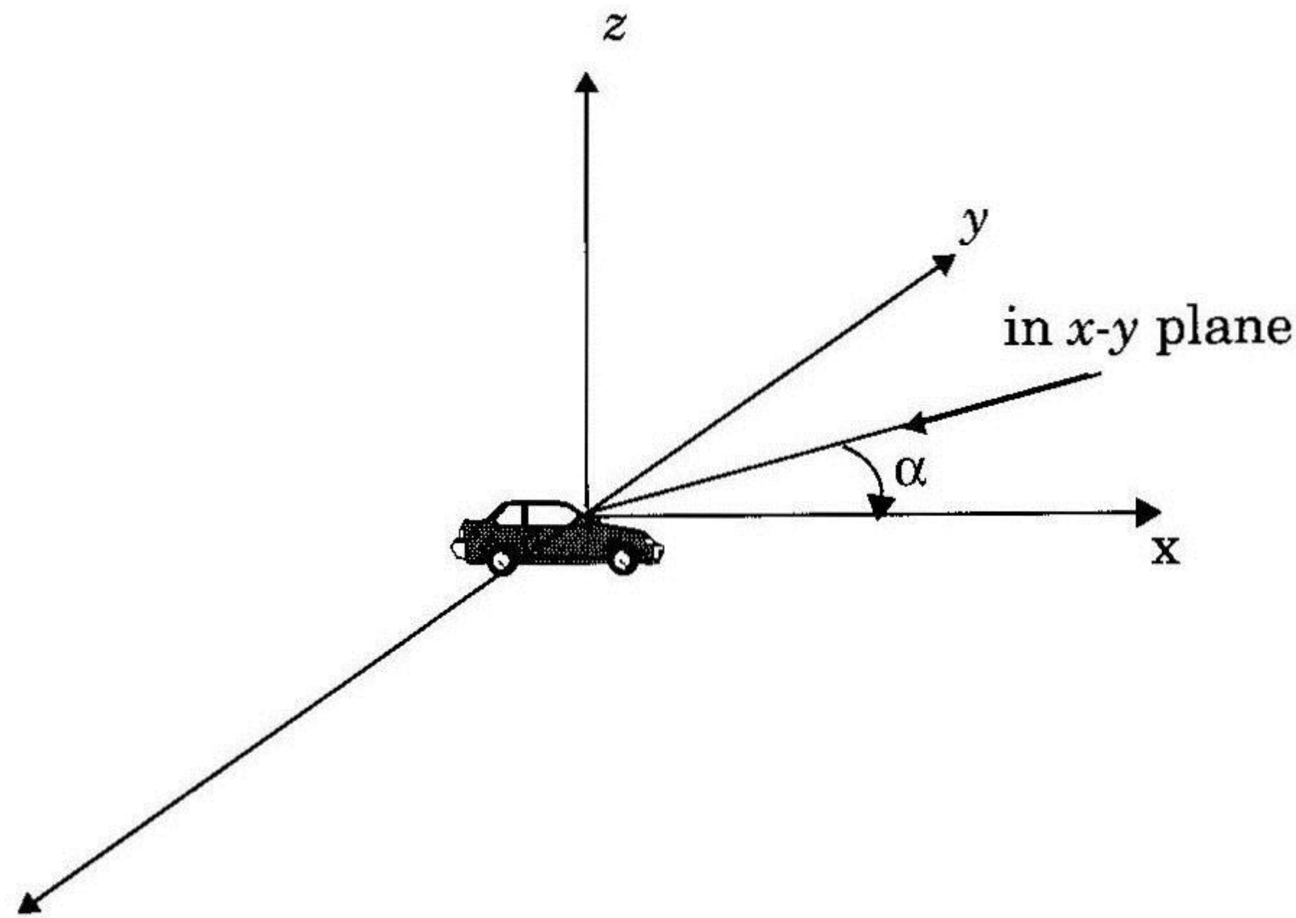


Figure 5.19 Illustrating plane waves arriving at random angles.

- Every wave arrives at the receiver at the same time, That is, no excess delay due to the multipath is assumed (flat fading assumption).
- For the nth wave arriving at an angle α_n to the x-axis, the Doppler shift in Hertz is given by

$$f_n = \frac{v}{\lambda} \cos \alpha_n$$

- The vertically polarized plane waves arriving at the mobile have *E* and *H* field components

$$E_z = E_n \sum_{n=1}^{N} C_n \cos(2\pi f_c t + \theta_n)$$

$$H_x = -\frac{E_0}{\eta} \sum_{n=1}^{N} C_n \sin\alpha_n \cos(2\pi f_c t + \theta_n)$$

$$H_y = -\frac{E_0}{\eta} \sum_{n=1}^{N} C_n \cos\alpha_n \cos(2\pi f_c t + \theta_n)$$

- The random phase θ_n is

$$\theta_n = 2\pi f_n t + \phi_n$$

and The amplitudes of the E- and H-field are normalized such that

$$\sum_{n=1}^{N} \overline{C_n^2} = 1$$

- Since the Doppler shift is very small when compared to the carrier frequency, the three field components may be modeled as narrow band random processes.
- If N is sufficiently large, the three components E_z, H_x, and H_y can be approximated as Gaussian random variables.

- Based on the analysis by Rice, the E-field can be expressed as

$$E_z(t) = T_c(t)\cos(2\pi f_c t) - T_s(t)\sin(2\pi f_c t)$$

where

$$T_c(t) = E_0 \sum_{n=1}^{N} C_n \cos(2\pi f_n t + \phi_n)$$

and

$$T_s(t) = E_0 \sum_{n=1}^{N} C_n \sin(2\pi f_n t + \phi_n)$$

- T_c and T_s are uncorrelated zero-mean Gaussian random variables with equal variance given by

$$\overline{T_c^2} = \overline{T_s^2} = \overline{|E_z|^2} = E_0^2 / 2$$

- The envelope of the received E-field is given by

$$|E_z(t)| = \sqrt{T_c^2(t) + T_s^2(t)} = r(t)$$

5.7.1.1 Spectral Shape Due to Doppler Spread in Clark's Model

- Gans develop a spectrum analysis for Clark's model.
- The total received power can be expressed as

$$P_r = \int_0^{2\pi} AG(\alpha)\,p(\alpha)\,d\alpha$$

- The instantaneous frequency of the received signal component arriving at an angle α is

$$f(\alpha) = f = \frac{v}{\lambda}\cos(\alpha) + f_c = f_m \cos\alpha + f_c$$

- We have

$$S(f)\left|df\right| = A\left[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)\right]\left|d\alpha\right|$$

$$\left|df\right| = \left|d\alpha\right|\left|-\sin\alpha\right| f_m$$

- And

$$\alpha = \cos^{-1}\left[\frac{f - f_c}{f_m}\right]$$

$$\sin\alpha = \sqrt{1-\left(\frac{f - f_c}{f_m}\right)^2}$$

- Then, the power spectral density $S(f)$ can be expressed as

$$S(f) = \frac{A[p(\alpha)G(\alpha) + p(-\alpha)G(-\alpha)]}{f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}$$

- For the case of a vertical $\lambda/4$ antenna, the output spectrum is given by

$$S_{E_z}(f) = \frac{1.5}{\pi f_m \sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}$$

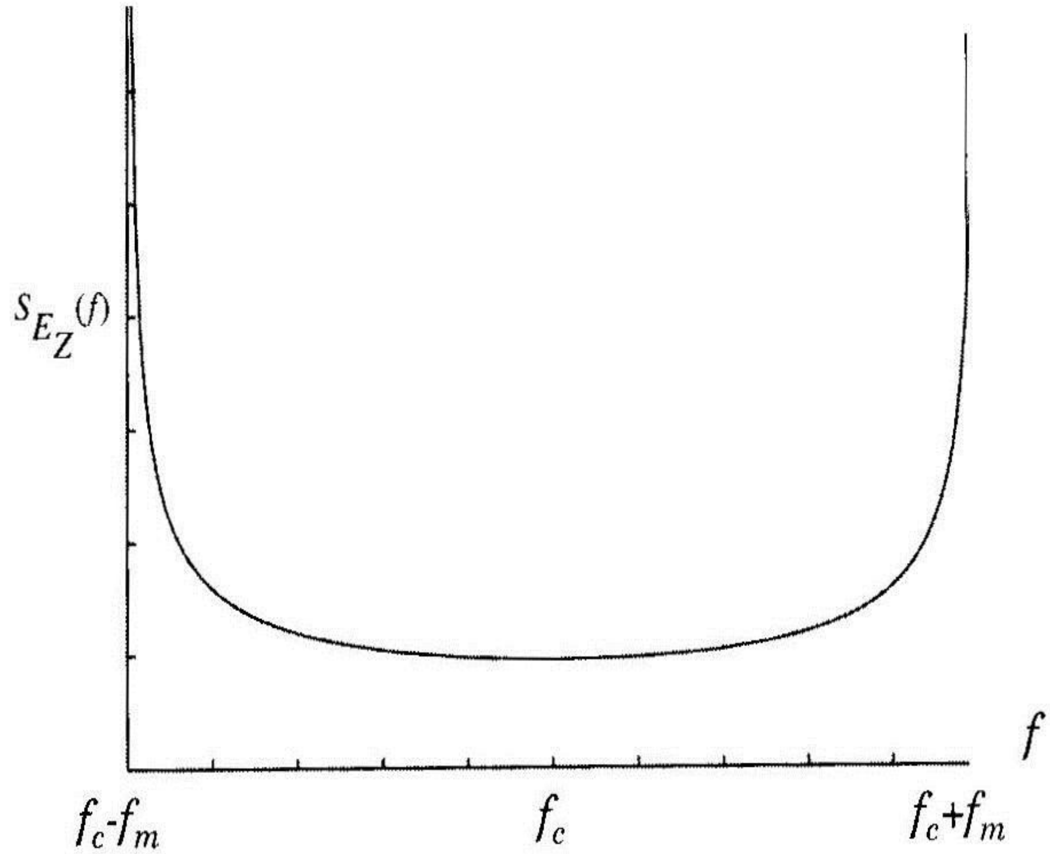


Figure 5.20 Doppler power spectrum for an unmodulated CW carrier [from [Gan72] © IEEE].

- After envelope detection of the Doppler-shift signal, the resulting baseband spectrum has a maximum frequency of $2f_m$

$$S_{bbE_Z}(f) = \frac{1}{8\pi f_m} K\left[\sqrt{1-\left(\frac{f}{2f_m}\right)^2}\right]$$

- Rayleigh fading simulators must use a fading spectrum such as Equation (5.78) in order to produce realistic fading waveforms that have proper time correlation.

5.7.2 Simulation of Clarke and Gans Fading Model

- Use the concept of in-phase and quadrature modulation paths to produce a simulated signal representing Equation (5.63)

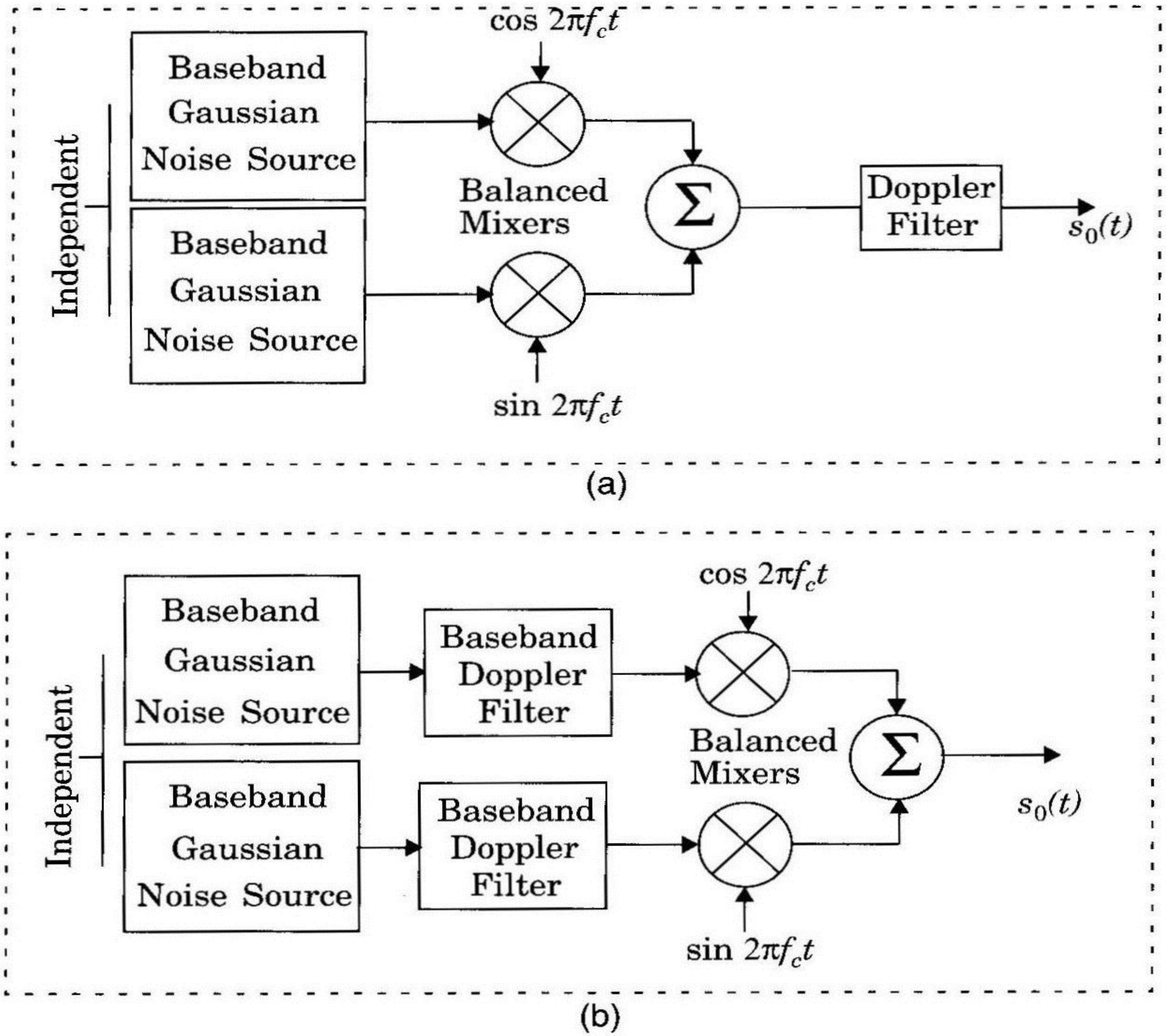


Figure 5.22 Simulator using quadrature amplitude modulation with (a) RF Doppler filter and (b) baseband Doppler filter.

- To handle the case of infinity at the passband edge, Smith truncated the value of $S_{E_z}(f_m)$ by using the slope of the function at the sample frequency just prior to the passband edge.

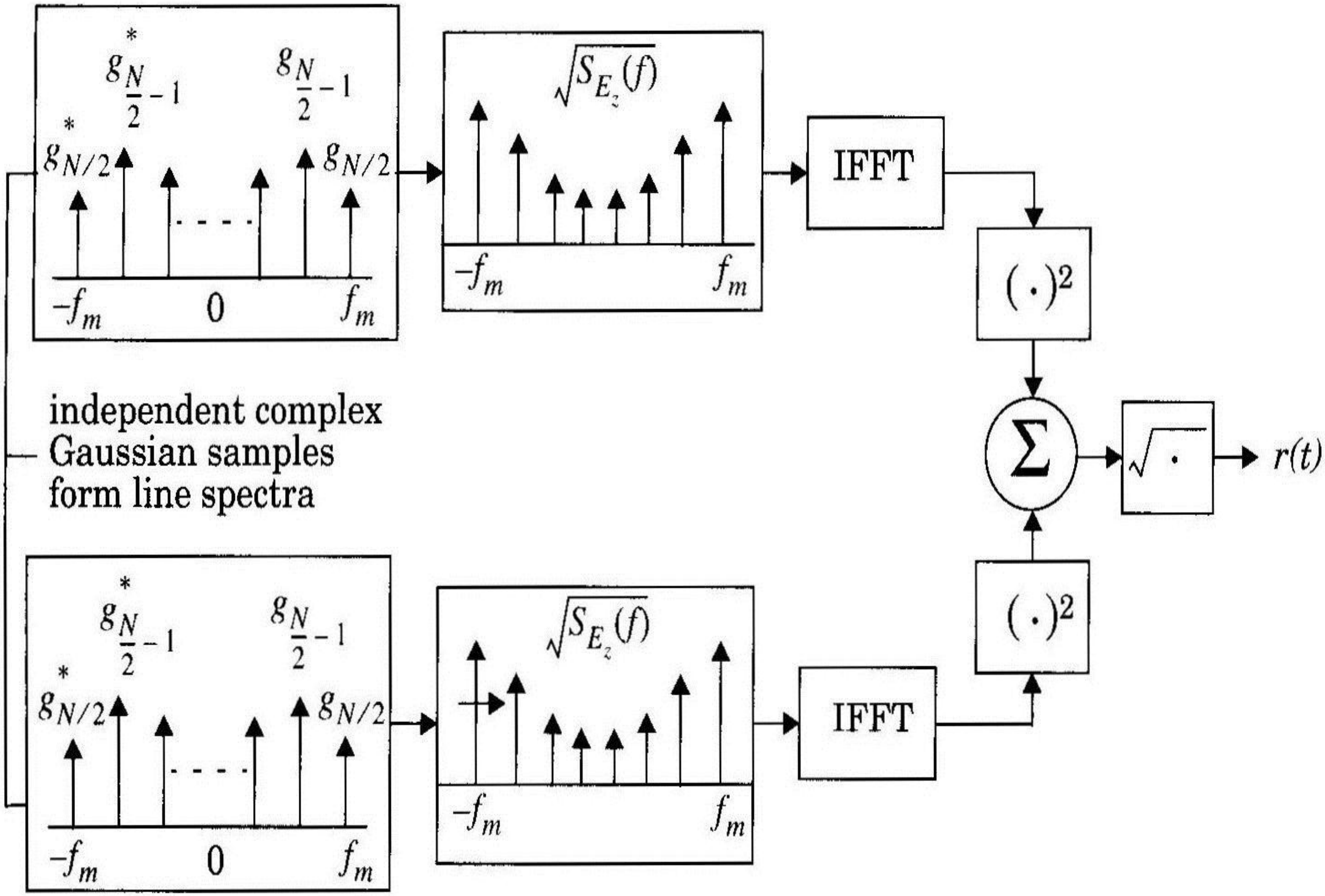


Figure 5.24 Frequency domain implementation of a Rayleigh fading simulator at baseband

1. Specify the number of frequency domain points N. (usually a power of two)
2. Compute the frequency spacing $\Delta f = 2f_m/(N-1)$
   This defines the time duration of a fading waveform, $T = 1/\Delta f$.
3. Generate complex Gaussian random variable for each of the $N/2$ positive frequency components of the noise source.

4. Construct the negative frequency component.
5. Multiply the fading spectrum $\sqrt{S_{E_z}(f)}$.
6. Perform FFT and add the squares of each signal point in time.
7. Take the square root of the sum to obtain an N-point time series of a simulated Rayleigh fading signal.

- By making a single frequency component dominant in amplitude within $\sqrt{S_{E_Z}(f)}$ and at $f=0$, the fading is changed from Rayleigh to Ricean.

- The frequency selective fading effects can be simulated by

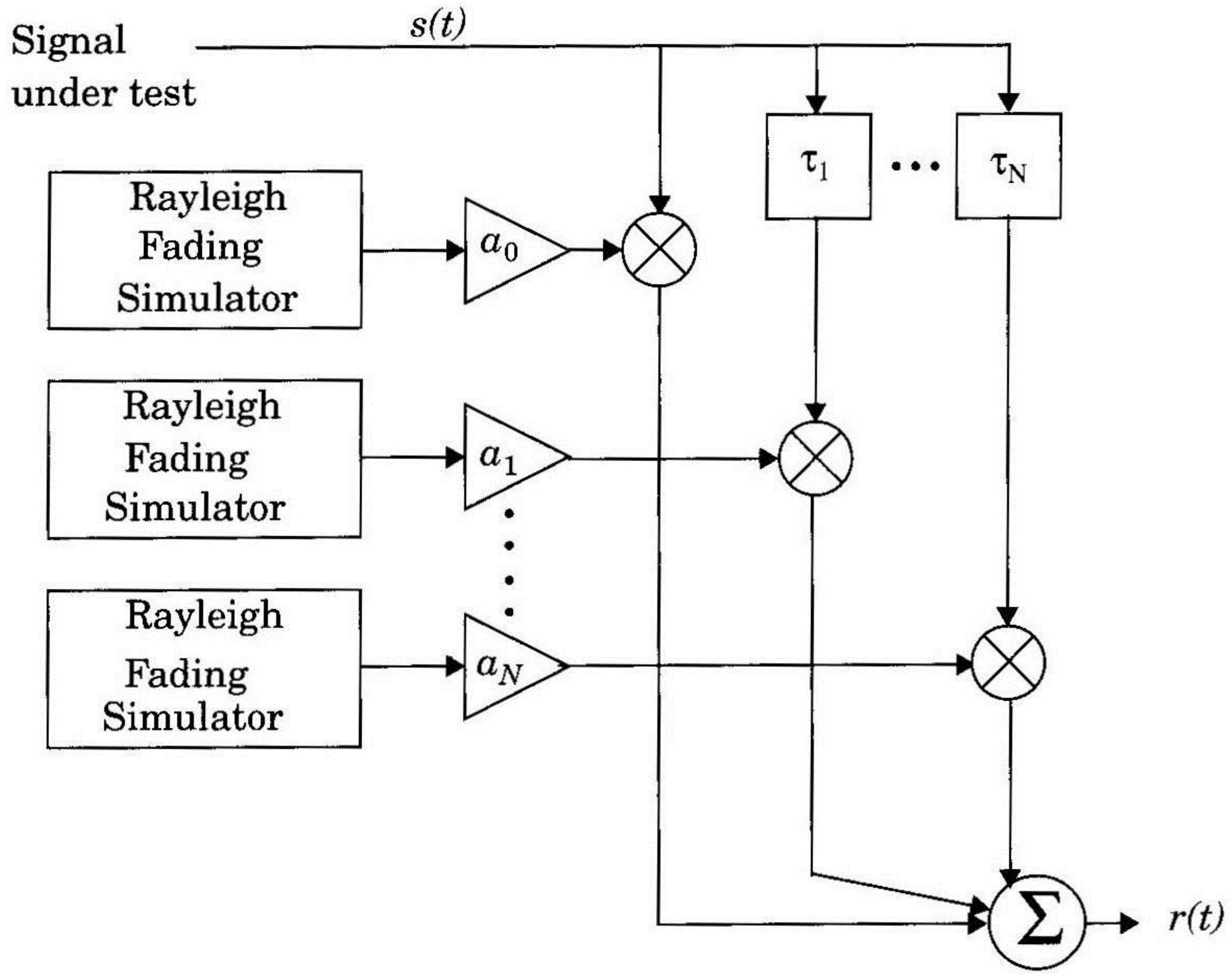


Figure 5.25 A signal may be applied to a Rayleigh fading simulator to determine performance in a wide range of channel conditions. Both flat and frequency selective fading conditions may be simulated, depending on gain and time delay settings.

- To determine the impact of flat fading on an applied signal $S(t)$, one merely needs to multiply the applied signal by $r(t)$.

5.7.3 Level Crossing and Fading Statistics

- The level crossing rate (LCR) and average fade duration of a Rayleigh fading signal are two important statistics which are useful for designing error control codes and diversity schemes.

- The level crossing rate (LCR) is defined as the expected rate at which the Rayleigh fading envelope crosses a specified level in a positive- going direction.

$$N_R = \int_0^\infty \dot{r}\ p(R,\dot{r})d\dot{r} = \sqrt{2\pi}\, f_m \rho e^{-\rho^2}$$

where $\dot{r}$ is the time derivative of $r(t)$ (i.e., the slope)

$p(R,\dot{r})$ is the joing probability density function

$\rho = R / R_{rms}$

- The maximum rate occurring at $\rho = 1/\sqrt{2}$, i.e., at a level 3dB below the rms level.

Example 5.7

For a Rayleigh fading signal, compute the positive-going level crossing rate for $\rho = 1$, when the maximum Doppler frequency (f_m) is 20 Hz. What is the maximum velocity of the mobile for this Doppler frequency if the carrier frequency is 900 MHz?

Solution

Using Equation (5.80), the number of zero level crossings is

$$N_R = \sqrt{2\pi}(20)(1)e^{-1} = 18.44 \text{ crossings per second}$$

The maximum velocity of the mobile can be obtained using the Doppler relation, $f_{d,\,max} = v/\lambda$.

Therefore velocity of the mobile at $f_m = 20$ Hz is

$$v = f_d\lambda = 20\text{ Hz}(1/3\text{ m}) = 6.66\text{ m/s} = 24\text{ km/hr}$$

- The average fade duration is defined as the average period of time for which the received signal is below a specified level *R*.
- For a Rayleigh fading signal, this is given by

$$\bar{\tau} = \frac{1}{N_R} Pr[r \le R]$$

since

$$Pr[r \le R] = \frac{1}{T}\sum_i \tau_i$$

then

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

Example 5.8

Find the average fade duration for threshold levels $\rho = 0.01$, $\rho = 0.1$, and $\rho = 1$, when the Doppler frequency is 200 Hz.

Solution

Average fade duration can be found by substituting the given values in Equation (5.84)

For $\rho = 0.01$, $\bar{\tau} = \frac{e^{0.01^2} - 1}{(0.01)200\sqrt{2\pi}} = 19.9\ \mu s$

For $\rho = 0.1$, $\bar{\tau} = \frac{e^{0.1^2} - 1}{(0.1)200\sqrt{2\pi}} = 200\ \mu s$

For $\rho = 1$, $\bar{\tau} = \frac{e^{1^2} - 1}{(1)200\sqrt{2\pi}} = 3.43$ ms

Example 5.9

Find the average fade duration for a threshold level of $\rho = 0.707$ when the Doppler frequency is 20 Hz. For a binary digital modulation with bit duration of 50 bps, is the Rayleigh fading slow or fast? What is the average number of bit errors per second for the given data rate. Assume that a bit error occurs whenever any portion of a bit encounters a fade for which $\rho < 0.1$.

Solution

The average fade duration can be obtained using Equation (5.84).

$$\bar{\tau} = \frac{e^{0.707^2} - 1}{(0.707)20\sqrt{2\pi}} = 18.3 \text{ ms}$$

For a data rate of 50 bps, the bit period is 20 ms. Since the bit period is greater than the average fade duration, for the given data rate the signal undergoes fast Rayleigh fading. Using Equation (5.84), the average fade duration for $\rho = 0.1$ is equal to 0.002 s. This is less than the duration of one bit. Therefore, only one bit on average will be lost during a fade. Using Equation (5.80), the number of level crossings for $\rho = 0.1$ is $N_r = 4.96$ crossings per seconds. Since a bit error is assumed to occur whenever a portion of a bit encounters a fade, and since average fade duration spans only a fraction of a bit duration, the total number of bits in error is 5 per second, resulting in a BER = (5/50) = 0.1.

5.7.4 Two-ray Rayleigh Fading Model

- A commonly used multipath model is an independent Rayleigh fading two-ray model.

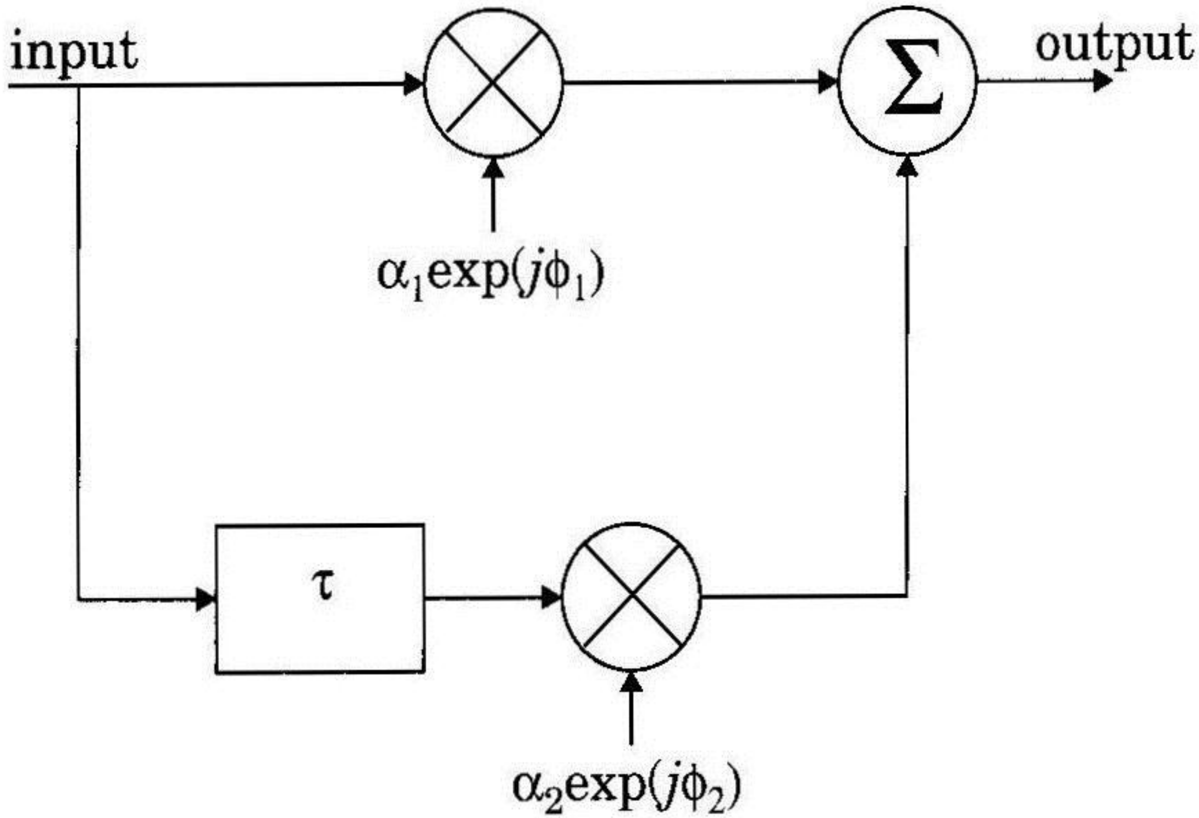


Figure 5.26 Two-ray Rayleigh fading model.

- The impulse response of the model is represented as

$$h_b(t) = \alpha_1 \exp(j\phi_1)\delta(t) + \alpha_2 \exp(j\phi_2)\delta(t-\tau)$$